DHANAMANJURI UNIVERSITY

Examination- 2024 (Dec)

Four year course B.A./B.Sc. 1^{st} Semester

Name of Programme : B.A./B.Sc. Mathematics
Paper Type : SEC(Theory)
Paper Code : SMA-001
Paper Title : Linear Programming and its Applications
Full Marks : 40
Pass Marks : 16 **Duration:2 Hours**

The figures in the margin indicate full marks for the questions.

1. Choose and rewrite the correct answer for each of the following: $1 \times 4 = 4$

a) The number of basic feasible solutions in a system of two linear equations in five variables will be
   i) 10 ii) at most 10
   iii) at most 15 iv) 15

b) Linear Programming problem (LPP) must have an
   i) Objective function that we need aim to maximize or minimize the function
   ii) Constraints that we need to specify the resources.
   iii) Decision variables that we need to determine.
   iv) All of the above.

c) Which of the following is not correct about LPP?
   i) all constraints must be linear relationships.
   ii) objective function must be linear.
   iii) all constraints and decision variables must be of either ' $\leq$ ' or '$\geq$' type.
   iv) all decision variables must be non-negative

d) The convex combination of two points x_1 and x_2 is a point x given by $x = \lambda_1 x_1 + \lambda_2 x_2$ such that $\lambda_1, \lambda_2 \geq 0$ and

i) $\lambda_1 + \lambda_2 = 2$ ii) $\lambda_1 + \lambda_2 = 1$

iii) $\lambda_1 + \lambda_2 \leq 1$ iv) $\lambda_1 + \lambda_2 \leq 2$

2. Answer the following questions: $1 \times 6 = 6$

a) Define a convex set.

b) Define a surplus variable.

c) Define an extreme point of a convex set.

d) What is meant by a feasible solution of an LPP ?

e) Define an artificial variable in solving LPP.

f) Define an optimal solution of an LPP.

3. Answer the following questions: $3 \times 4 = 12$

a) Write down the LPP in standard form:

Maximize $Z = 3x_1 + 2x_2 + 5x_3$

subject to constraints:

$$2x_1 - 3x_2 \leq 3$$
$$x_1 + 2x_2 + 3x_3 \geq 5$$
$$3x_1 + 2x_2 \leq 2$$
$$x_1 \geq 0, x_2 \geq 0 \text{ and } x_3 \geq 0$$

b) Find all the basic solutions of

$$x_1 + 2x_2 + x_3 = 4$$
$$2x_1 + x_2 + 5x_3 = 5$$

c) Prove that the intersection of two convex sets is also a convex set.

d) An oil company has two units A and B which produces three different grades of oil super fine, medium and low grade oil. The company has to supply 12,8 and 24 barrels of superfine, medium and low grade oils respectively per week. It costs the company Rs. 1,000 and Rs. 800 per day to run the units A and B respectively. On a day Unit A produces 6,2 and 4 barrels and the unit B produces 2,2 and 12 barrels of super fine,medium and low grade oil per day. The manager has to decide on how

many days per week should each unit be operated in order to meet the requirement at minimum cost. Formulate the LPP model.

4. Answer any two of the following questions: 9 ×2 = 18

a) An industry is manufacturing two types of products A and B. The profits per Kg of the two products are Rs 30 and Rs 40 respectively. These two products require processing in three types of machines. The following table shows the available machine hours per day and the time required on each machine to produce one Kg of A and B.

Profit per Kg	**A**	**B**	**Available hours per day**
Machine 1	3	2	600
Machine 2	3	5	800
Machine 3	5	6	1100

Formulate the linear programming model and solve graphically.

b) Write the algorithm for solving an LPP by using simplex method.

c) Use two phase Simplex method to solve the following LPP:
$Maximize \qquad Z = -x_1 - x_2$
subject to the constraints

$$2x_1 + x_2 \geq 4$$
$$x_1 + 7x_2 \geq 7$$
$$x_1, x_2 \geq 0$$

d) Use Big M method to solve the following LPP:
$Maximize \qquad Z = 6x_1 + 4x_2$
subject to constraints

$$2x_1 + 3x_2 \leq 30$$
$$3x_1 + 2x_2 \leq 24$$
$$x_1 + x_2 \geq 3$$
$$x_1, x_2 \geq 0$$

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