DHANAMANJURI UNIVERSITY

Examination- 2024 (June)

M.Sc. 2^{nd} Semester

Name of Programme	**:**	**M.Sc. Mathematics**
Paper Type	**:**	**Theory**
Paper Code	**:**	**MAT-510**
Paper Title	**:**	**Differential Equations (Partial)-II**

Full Marks : 40

Pass Marks : 16 **Duration: 2 Hours**

The figures in the margin indicate full marks for the questions.

Answer any four of the following questions: **$10 \times 4 = 40$**

1. Find the equation of the system of surfaces which cut orthogonally the cones of the system $x^2+y^2+z^2=cxy$ and also the particular surface which passes through the circle $x^2+y^2=1, z=3$.

2. Determine the characteristics of the equation $z=p^2-q^2$ and find the integral surface which passes through the parabola $4z+x^2=0, y=0$.

3. Show that the equations $xp=yq$ and $z(xp+yq)=2xy$ are compatible and solve them.

4. Prove that if $(a_rD+b_rD'+c_r)^n$ $(a_r\neq 0)$ is a factor of $F(D,D')$ and if the functions $\phi_{r_1},\phi_{r_2},...,\phi_{r_n}$ are arbitrary, then $e^{-\frac{c_r}{a_r}x}\sum_{s=1}^{n}x^{s-1}\phi_{rs}(a_ry-b_rx)$ is a solution of $F(D,D')z=0$.

5. Solve:

   a) $ys-p=xy^2\ \cos(xy)$

   b) $s-t=\dfrac{x}{y^2}$

6. Solve the equation $r-t\cos^2 x+p\tan x=0$ by Monge's method.

7. Let a thin homogeneous string which is perfectly flexible under uniform tension lie in its equilibrium position along the x-axis. The ends of the string are fixed at $x = 0$ and $x = L$. The string is pulled aside a short distance and released. If no external forces are present which correspond to the case of free vibrations, obtain the solution $u(x,t)$ of the IBVP which describes the motion of the vibrating string.

8. Find the general solution of the Neumann problem for a rectangle defined as follows:
   PDE: $\nabla^2 u = 0, \quad 0 \le x \le a, \quad 0 \le y \le b$
   BCs: $u_x(0,y) = 0, \quad u_x(a,y) = 0, \quad u_y(x,0) = 0, \quad u_y(x,b) = f(x)$

*****