DHANAMANJURI UNIVERSITY

Examination- 2024 (June)

M.Sc. 2nd Semester

Name of Programme : M.Sc. Mathematics
Paper Type : Theory
Paper Code : MAT-509
Paper Title : Complex Analysis-II
Full Marks : 40
Pass Marks : 16 Duration: 2 Hours

The figures in the margin indicate full marks for the questions.

Answer any four of the following questions: $10 \times 4 = 40$

1. When is a family of functions said to be Normal? Prove that a family F in $H(G)$ is normal iff F is locally bounded.

2. Define Elementary Factor. If $|z| \leq 1$ and $p \geq 0$, then prove that $|1 - E_p(z)| \leq |z|^{p+1}$.

3. Prove that $\Gamma(z) = \lim_{n\to\infty} \frac{n!\, n^z}{z(z+1)\cdots(z+n)}$, where symbols have their usual meanings. Hence show that $\Gamma(z+1) = z\Gamma(z)$.

4. Establish the formula $\log|f(0)| = -\sum_{i=1}^{n} \log\left(\frac{R}{|z_i|}\right) + \frac{1}{2\pi}\int_0^{2\pi} \log f(Re^{i\Phi})d\Phi$ for an analytic function f in the disk $|z| \leq R$.

5. State and prove Hadamard's Three Circle Theorem.

6. Define Order of an analytic function. Find the order of the function $f(z) = \cos z$.

7. Let f be an analytic function on a region containing the closure of the disk $D = \{z : |z| < 1\}$ and satisfying $f(0) = 0$, $f'(0) = 1$. Then prove that there is a disk $S \subset D$ on which f is one-one and such that $f(S)$ contains a disk of radius $\frac{1}{72}$.

8. Define Landau's constant stating its range or values. If f is an entire function that omits two values, then prove that f is a constant.

*****