DHANAMANJURI UNIVERSITY

Examination- 2024 (June)
M.Sc. 2nd Semester

Name of Programme : M.Sc. Mathematics
Paper Type : Theory
Paper Code : MAT-508
Paper Title : Topology-II
Full Marks : 40
Pass Marks : 16 Duration: 2 Hours

The figures in the margin indicate full marks for the questions.

Answer any four of the following questions: $10\times4=40$

1. **Answer the following questions:**

   a) Define a Topological space. 5

   b) Prove that a non-empty subset of the real line is connected if and only if it is an interval. 5

2. State T_0, T_1 and Hausdorff space. Prove that the product space of two

   a) T_0 spaces is a T_0 space.

   b) T_1 spaces is a T_1 space.

   c) Hausdorff spaces is a Hausdorff space. 10

3. State and prove Urysohn Metrisation Theorem. 10

4. a) A space X is Hausdorff iff each net in X converges to atmost one point in X. 5

   b) Let X be a Hausdorff space. Prove that

      i) A compact subset of X is closed.

      ii) Any two disjoint compact subsets of X have disjoint nbds. 5

5. a) Prove that a T_1 space X is countably compact iff it has the Bolzano-Weierstrass property. 5

   b) Prove that if X and Y are compact, then so is $X \times Y$. 5

6. State and prove the Fundamental Theorem of Algebra. 10

7. Define Pseudo-metric. State and prove Nagata-Smirnov metrisaton theorem. 10

8. State paracompact. Prove that the product of a paracompact space and a compact space is paracompact. 10

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