DHANAMANJURI UNIVERSITY

Examination- 2024 (June)
M.Sc. 2^{nd} Semester

Name of Programme	**:**	**M.Sc. Mathematics**
Paper Type	**:**	**Theory**
Paper Code	**:**	**MAT-506**
Paper Title	**:**	**Advanced Abstract Algebra- II**

Full Marks : 40

Pass Marks : 16 **Duration: 2 Hours**

The figures in the margin indicate full marks for the questions.

Answer any four of the following questions: **10×4=40**

1. If $0 \to M' \xrightarrow{f} M \xrightarrow{g} M'' \to 0$ is a split exact sequence with splitting $t : M' \to M$, then prove that $M = f(M') \oplus t(M'') \cong M' \oplus M''$. 10

2. a) Prove that $P = \oplus \sum_\alpha P_\alpha$ is projective iff P_α is projective for each α. 6

   b) Show that every free module is projective. Is the converse true? Justify it. 4

3. a) For an R-module M, prove that the following are equivalent

      i) M is Artinian.

      ii) every quotient module of M is finitely cogenerated.

      iii) every non empty set of submodules of M has a minimal element. 6

   b) Prove that every submodule and homomorphic image of a Noetherian module is Noetherian. 4

4. In a Noetherian ring R that has no non zero nilpotent ideals, show that there is no non zero nil ideal. 10

5. State and prove Hilbert Basis theorem. 10

6. Find the invariant factors and Jordan Canonical form of the matrix A over $\mathbb{Q}$, where the matrix A is given by $A = \begin{bmatrix} 0 & 4 & 2 \\ -3 & 8 & 3 \\ 4 & -8 & 2 \end{bmatrix}$ 10

7. Let $M = \sum_{\alpha \in \Lambda} M_\alpha$ be a sum R-modules M_α, and K be a submodule of M. Then prove that there exists a subset Λ' of Λ such that $\sum_{\sum_{\alpha \in \Lambda'}} M_\alpha$ is a direct sum, and $M = K \oplus \left(\oplus \sum_{\alpha \in \Lambda'} M_\alpha\right)$. 10

8. a) Let M be a simple R-module. Then show that $Hom_R(M, M)$ is a division ring.

   b) Let $M = M_1 \oplus M_2$ be such that M_1 and M_2 are simple $M_1 \not\cong M_2$. Show that the ring $End_R(M)$ is a sum of division ring. 10

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