DHANAMANJURI UNIVERSITY

Examination- 2024 (December)

M.Sc.1 st Semester

Name of Programme : M.Sc. Mathematics
Paper Type : Theory
Paper Code : MAT-503
Paper Title : Topology-I
Full Marks : 80
Pass Marks : 32 **Duration: 3 Hours**

The figures in the margin indicate full marks for the questions.

Answer all the questions:

1. a) Define 5

   i) Topological space
   ii) Discrete metric space
   iii) interior point
   iv) exterior point
   v) boundary point.

   b) Let X be a space and let B be a base. Then show that B has the following properties: 5

   i) $X = \cup\{B : B \in \mathcal{B}\}$
   ii) For each $B_1, B_2 \in \mathcal{B}$ and every $x \in B_1 \cap B_2$, there exists a $B \in \mathcal{B}$ such that $x \in B \subseteq B_1 \cap B_2$.

2. a) Show that Euclidean space $\square^n$ is a metric space with the metric defined by

$$d(x, y) = \sqrt{(\xi_1 - \eta_1)^2 + (\xi_2 - \eta_2)^2 + \cdots + (\xi_n - \eta_n)^2}$$

6

   b) Does $d(x, y) = (x - y)^2$ define a metric on the set of all real numbers and why? 4

3. a) Prove that a function $f : X \to Y$ is continuous if and only if it continuous at each point of X. 5

   b) State and prove Lindelof's Theorem. 5

4. State and prove Urysohn's Lemma. 10

5. a) Let X be a Hausdorff space. Prove that 6

i) A compact subset of X is closed

ii) Any two disjoint compact subsets of X have disjoint nbds.

b) Prove that a locally compact subspace of a Hausdorff space is locally closed. 4

6. a) If $(X, \mathfrak{J})$ is a topological space and $Y \subseteq X$ and $\mathfrak{J}_Y = \{G \cap Y : G \in \mathfrak{J}\}$, then prove that $\mathfrak{J}_Y$ is a topology on Y. 5

b) Let A be a subset of space X. Prove that $\overline{A} = A \cup A'$. 5

7. a) Let X be space and $A, B \subseteq X$. Show that 6

i) $\overline{\overline{A}} = \overline{A}$

ii) $A \subseteq B \Rightarrow \overline{A} \subseteq \overline{B}$

iii) $\overline{A \cup B} = \overline{A} \cup \overline{B}$

iv) $\overline{A \cup B} \subseteq \overline{A} \cap \overline{B}$

b) Show that intersection of two topology is again a topology. 4

8. a) Let X and Y be spaces and $f : X \to Y$ a function. Prove that f is closed if and only if $\overline{f(A)} \subseteq f(\overline{A})$ for each set $A \subseteq X$. 5

b) Prove that every basis of a second countable space contains a countable subfamily which is also a basis. 5

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