DHANAMANJURI UNIVERSITY
DECEMBER 2025

Name of Programme : **B.A./B.Sc. Mathematics**
Semester : **5ᵗʰ**
Paper Type : **Core**
Paper Code : **CMA-313**
Paper Title : **Multivariate Calculus**
Full Marks : **80**
Pass Marks : **32**   **Duration: 3 Hours**

The figures in the margin indicate full marks for the corresponding questions.
Answer all questions:

1. Choose the correct answer from the following and rewrite it :   **1 × 3 = 3**

a) If $x = r\cos\theta$ and $y = r\sin\theta$, then the value of $\dfrac{\partial(x,y)}{\partial(r,\theta)}$ is

   i) r

   ii) $\dfrac{1}{r}$

   iii) 0

   iv) $\dfrac{1}{r^2}$

b) The stationary point of $f(x,y) = y^2 + 4xy + 3x^2 + x^3$

   i) $\left(-\dfrac{2}{3}, -\dfrac{4}{3}\right)$

   ii) $\left(\dfrac{2}{3}, -\dfrac{4}{3}\right)$

   iii) $\left(-\dfrac{2}{3}, \dfrac{4}{3}\right)$

   iv) $\left(\dfrac{2}{3}, \dfrac{4}{3}\right)$

c) The Stoke's Theorem is a relation between

   i) line integral and double integral

   ii) line integral and surface integral

   iii) line integral and volume integral

   iv) surface integral and volume integral

2. Write very short answers for each of the following questions.   **1 × 6 = 6**

a) Find $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy+2}{x^2+y^2}$.

b) State Green's theorem.

c) Find $\displaystyle\int_{-1}^{1} \int_{-2}^{2} \int_{-3}^{3} dx\, dy\, dz$.

d) If (x, y, z) and (z, ρ, ϕ) are the Cartesian and cylindrical coordinates of a point P, then write the value of $dx\, dy\, dz$ in terms of cylindrical coordinates.

e) Define volume integral.

f) Find $\dfrac{\partial u}{\partial r}$ if $u = e^{r\cos\theta}.\cos(r\sin\theta)$.

3. Write short answer for each of the following questions.   **3 × 5 = 15**

a) If $u = u(y - z, z - x, x - y)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.

b) If $\phi = 3x^2 y - y^3 z^2$, find $\mathrm{grad}\phi$ at the point $(1, -2, -1)$.

c) Evaluate $\displaystyle\int_0^\pi \int_0^{a(1-\cos\theta)} r^2 \sin\theta\, dr\, d\theta$.

d) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at $(2, -1, 2)$.

e) If a force $\vec{F} = 2x^2 y\hat{i} + 3xy\hat{j}$ displaces a particle in the xy-plane from $(0, 0)$ to $(1, 4)$ along a curve $y = 4x^2$. Find the work done.

4. Answer the following questions: **4×5 = 20**

a) Discuss the continuity of $f(x, y) = \begin{cases} \dfrac{x}{\sqrt{x^2+y^2}}, & x \neq 0, y \neq 0 \\ 2, & x = 0, y = 0 \end{cases}$

b) If $f(x, y) = \begin{cases} \dfrac{x^2-xy}{x+y}, & x \neq 0, y \neq 0 \\ 0, & x = 0, y = 0 \end{cases}$, find $f_x(0, 0)$, $f_y(0, 0)$.

c) Find the directional derivative of $\phi = 4e^{2x-y+z}$ at the point $(1, 1, 1)$ in the direction towards the point $(-3, 5, 6)$.

d) Evaluate $\int_0^{\pi/2} \left[\int_0^{a \cos \theta} r \sqrt{a^2 - r^2} \, dr \right] d\theta.$

e) Find the divergence and curl of $\vec{V} = xyz\hat{i} + 3x^2y\hat{j} + (xz^2 - y^2z)\hat{k}$ at $(2, -1, 1)$.

5. Answer any two questions: $\qquad$ **6 × 2 = 12**

a) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right)^2 u = -\frac{9}{(x+y+z)^2}.$

b) Show that the minimum value of $f(x, y) = xy + a^3 \left(\frac{1}{x} + \frac{1}{y} \right)$ is $3a^2$.

c) Find the dimension of rectangular box of maximum capacity whose surface area is given when (a) box is open at top and (b) box is closed.

6. Answer any two questions: $\qquad$ **6 × 2 = 12**

a) Evaluate $\int_0^{\log 2} \int_0^x \int_0^{x+y} e^{x+y+z} \, dz \, dy \, dx.$

b) Change the order of integration and evaluate $\int_0^a \int_0^x \frac{dy \, dx}{(y + a)\sqrt{(a - x)(x - y)}}.$

c) Find the volume bounded by the paraboloid $x^2 + y^2 = az$ and the cylinder $x^2 + y^2 = a^2$.

7. Answer any two questions: $\qquad$ **6 × 2 = 12**

a) Evaluate $\iint_S \vec{A} \cdot \hat{n} \, dS$, where $\vec{A} = 18z\hat{i} - 12\hat{j} + 3y\hat{k}$ and S is part of the plane $2x + 3y + 6z = 12$ included in the first octant.

b) Apply Green's theorem to evaluate $\int_C [(2x^2 - y^2) \, dx + (x^2 + y^2) \, dy]$ where C is the boundary of the area enclosed by the x-axis and the upper half of circle $x^2 + y^2 = a^2$.

c) Use Gauss's divergence theorem to show that
$\iint_S \nabla(x^2 + y^2 + z^2) \, d\vec{S} = 6V$ where S is any closed enclosing volume V.

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