# DHANAMANJURI UNIVERSITY
# DECEMBER 2025

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| **Name of Programme** | **:** | **B.A./B.Sc. Mathematics** |
| **Semester** | **:** | **1st** |
| **Paper Type** | **:** | **Core** |
| **Paper Code** | **:** | **CMA-102** |
| **Paper Title** | **:** | **Algebra** |

**Full Marks : 80**

**Pass Marks : 32** **Duration: 3 Hours**

***The figures in the margin indicate full marks for the corresponding questions.***
***Answer all questions:***

**1. Choose and rewrite the correct answer for each of the following:** **1×3 = 3**

(a) The exponential value of $\sinh x$ is

$(i)\dfrac{e^{ix}-e^{-ix}}{2i}$ (ii) $\dfrac{e^{ix}+e^{-ix}}{2i}$

(iii) $\dfrac{e^{x}-e^{-x}}{2}$ (iv) $\dfrac{e^{x}+e^{-x}}{2}$

(b) If $a$, $b$, $c$ are in harmonic progression, then

(i) $b=\dfrac{a+c}{2}$ (ii) $b=\dfrac{2ac}{a+c}$

(iii) $b=\dfrac{a+c}{2ac}$ (iv) $b=\dfrac{ac}{2(a+c)}$

(c) For every characteristic root of a Matrix,

(i) G.M. $>$ A.M. (ii) G.M. $\geq$ A.M.

(iii) G.M. $\leq$ A.M. (iv) G.M. $=$ A.M.

where G.M., A.M. is geometric multiplicity and algebraic multiplicity respectively.

**2. Write very short answer for each of the following questions:** **1 × 6 = 6**

(a) State De Moivre's theorem.

(b) Show that $\cosh x = \cos ix$.

(c) State fundamental theorem of algebra.

(d) Write a matrix which is symmetric but not Hermitian.

(e) What will be the characteristic roots of a Hermitian matrix?

(f) What will be rank of a matrix when its row rank is 2?

**3. Write short answer for each of the following questions:** **$3 \times 5 = 15$**

(a) Find the principal value of $\cosh^{-1} z$.

(b) Show that the equation $x^4 - 2x^3 - 1 = 0$ has two imaginary roots.

(c) Find the equation whose roots are equal to the roots of $x^3 - 9x^2 + 28x - 27 = 0$ each diminished by 3.

(d) Find the rank of the matrix

$$\begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 4 & 6 & 2 \\ 1 & 2 & 3 & 2 \end{bmatrix}$$

by reducing to Echelon form.

(e) Show that the vectors $[2\ 3\ -1\ -1]$, $[1\ -1\ -2\ -4]$, $[3\ 1\ 3\ -2]$, $[6\ 3\ 0\ -7]$ are linearly dependent.

**4. Write answer for each of the following questions:** **$4\times5 = 20$**

(a) If $a = \cos\theta + i\sin\theta$, $b = \cos\varphi + i\sin\varphi$, find the values $\cos(\theta+\varphi)$ and $\cos(\theta-\varphi)$ in terms of $a, b$.

(b) Evaluate $\mathrm{Log}(\alpha + i\beta)$, where $\alpha$ and $\beta$ are real.

(c) If $x > 0$, $y > 0$, $z > 0$ and $x + y + z = 1$, prove that

$$\frac{x}{2-x} + \frac{y}{2-y} + \frac{z}{2-z} \geq \frac{3}{5}.$$

(d) State and prove Holder's inequality.

(e) Show that every square matrix $A$ can be expressed uniquely as $P + iQ$ where $P$ and $Q$ are Hermitian matrices.

**5. Answer any two of the following questions:** **$6\times2 = 12$**

(a) Find all the values of $\sqrt[3]{1+i}$ .

(b) State and prove Gregory's series.

(c) Find the sum of the infinite series

$$\sin\alpha + \frac{1}{2}\sin 2\alpha + \frac{1}{2^2}\sin 3\alpha + \cdots$$

**6. Answer any two of the following questions:** **6×2 = 12**

(a) Solve the reciprocal equation $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$.

(b) Solve the biquadratic equation $x^4 - 10x^2 + 4x + 8 = 0$ by using Ferrari's method.

(c) Solve the equation $9x^3 - 6x^2 + 1 = 0$ by using Cardan's method.

**7. Answer any two of the following questions:** **6×2 = 12**

(a) Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 4 & 5 & 4 \end{bmatrix}$$

by using the matrix equation $AX = B$.

(b) Find the Latent values of the matrix

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

Find also the Latent vector/vectors corresponding to the smallest Latent value.

(c) Find the characteristic equation of the matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}.$$

Verify Cayley-Hamilton theorem for this matrix and hence obtain $A^{-1}$.

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