

DHANAMANJURI UNIVERSITY

DECEMBER 2025

Name of Programme : B.A./B.Sc. Mathematics
Semester : 1st
Paper Type : Core
Paper Code : CMA-101
Paper Title : Calculus
Full Marks : 80
Pass Marks : 32 **Duration: 3 Hours**

The figures in the margin indicate full marks for the corresponding questions.

Answer all questions:

1. Choose and rewrite the correct answer for each of the following: **1×3 = 3**

a) The limit of the function $e^x + e^{-x}$ when $x \rightarrow 0$ is

- i. 1
- ii. 0
- iii. 2
- iv. -1

b) The formula for the radius of curvature of a plane curve $y = f(x)$ is

- i. $\frac{\left(1 + \frac{dy}{dx}\right)^{3/2}}{\frac{d^2y}{dx^2}}$
- ii. $\frac{\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{1/2}}{\frac{d^2y}{dx^2}}$
- iii. $\frac{\left(1 + \frac{dy}{dx}\right)^{3/2}}{\frac{d^2y}{dx^2}}$
- iv. $\frac{\left(1 + \left(\frac{dx}{dy}\right)^2\right)^{1/2}}{\frac{d^2y}{dx^2}}$

c) The length of the arc of the curve along y -axis between $y = c$ and $y = d$ is

$$\text{i. } s = \int_c^d \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$\text{ii. } s = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$$

$$\text{iii. } s = \int_c^d \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dy$$

$$\text{iv. } s = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dx$$

2. Write very short answer for each of the following questions:

1×6=6

- State Rolle's theorem.
- What is the curvature of a curve at a point?
- Define a point of inflexion of a curve.
- Find the volume of whole sphere generated by revolving a circle $x^2 + y^2 = a^2$ about x - axis.
- Write the formula to calculate the surface area of solid formed by rotating the curve $y = f(x)$ about x - axis.
- Write a reduction formula for $\int_0^{\frac{\pi}{2}} \sin^m x \cos^n x dx$.

3. Write short answer for each of the following:

3×5=15

- Find n^{th} derivative of $y = \cos 2x \cos x$.
- Find all the relative extreme of the function $f(x) = x^3 - 3x$ by using the first derivative test.
- Verify Euler's theorem for the function $u = x^3 + y^3 + 3x^2 + 3xy^2$.
- Find the relative maximum and relative minimum of the curve $f(x) = x^3 - 9x^2 + 5x + 7$ using 2^{nd} derivative test.
- Evaluate $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2) dy dx$.

4. Write short answer for each of the following questions:**4×5=20**

- a) Expand the function e^x in a finite series in power of x , with the remainder in Cauchy's form.
- b) Show that the function $f(x, y) = x^2 + 2xy + y^2 + x^3 + y^3 + x^7$ has neither a maximum nor minimum at the origin.
- c) Find the radius of curvature at any point (r, θ) for the curve $r^2 \cos 2\theta = a^2$.
- d) Examine the curve $y = x^3 - 9x^2 + 7x$, regarding its convexity or concavity. Determine its points of inflexion.
- e) Find the area of the quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ between the major and minor axes.

5. Answer any two questions:**6×2=12**

- a) State and prove Lagrange's Mean Value theorem.
- b) If $y = \sin^{-1} x$, then show that
- i. $(1 - x^2)y_2 - xy_1 = 0$
 - ii. $(1 - x^2)y_{n+2} - 2(n + 1)xy_{n+1} - n^2y_n = 0$.
- c) Evaluate $\lim_{x \rightarrow 0} (\cos x)^{\cos^2 x}$.

6. Answer any two questions:**6×2=12**

- a) Find the asymptotes of the curve

$$y^3 - 6xy^2 + 11x^2y - 6x^3 + y^2 - x^2 + 2x - 3y - 1 = 0.$$

- b) State and prove converse of Euler's theorem on Homogeneous functions of three variables.
- c) If $u = \sin^{-1} \frac{x}{y} + \tan^{-1} \frac{x}{y}$, show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0$.

7. Answer any two questions:**6×2=12**

- a) Find the length of the curve of the perimeter of the astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$.
- b) Obtain reduction formula for $\int \sin^n x dx$ and hence find the reduction formula for $\int_0^{\frac{\pi}{2}} \sin^n x dx$, where n is a positive integer. Hence use this formula to evaluate $\int_0^{\frac{\pi}{2}} \sin^7 x dx$.
- c) Find the volume and surface area of the solid obtained by revolving the circle $x^2 + y^2 = a^2$ about x -axis.