CMA-318

DHANAMANJURI UNIVERSITY

Examination- 2026 (June)

Name of Programme	:	**B.A./ B.Sc. Mathematics**
Semester	:	**6^{th}**
Paper Type	:	**Core**
Paper Code	:	**CMA-318**
Paper Title	:	**Probability Theory and Statistics**

Full Marks : 80

Pass Marks : 32

Duration: 3 Hours

The figures in the margin indicate full marks for the questions.

Answer all the questions:

1. Choose the correct one and rewrite it from the following: $1 \times 3 = 3$

a) If X is a random variable then which of the following gives the characteristic function of X?

i) $E(X)$
ii) $E(cX)$
iii) $E(e^{tX})$
iv) $E(e^{itX})$

b) Which one is the value of variance of exponential distribution $f(x,\theta) = \begin{cases} \theta e^{-\theta x}, & x \geq 0 \\ 0, & otherwise \end{cases}$?

i) $\frac{1}{\theta}$
ii) $\frac{1}{\theta^2}$
iii) θ^2
iv) θ

c) Which of the following statement is true?

i) Coefficient of correlation depends either on change of origin or on the change of scale.
ii) Coefficient of correlation depends on change of origin only.
iii) Coefficient of correlation depends on change of scale only.
iv) Coefficient of correlation depends neither on change of origin nor on the change of scale.

2. Write very short answer for each of the following questions. $1 \times 6 = 6$

a) Does $f(x)$ represent probability density function if

$$f(x) = \begin{cases} \frac{1}{2}(3-x), 0 < x < 1 \\ \frac{1}{2}(3+x), 1 \le x < 2 \end{cases} ?$$

b) Write down the formula for effect of change of origin and scale of characteristic function.

c) Represent Normal distribution diagrammatically.

d) Subway trains on a certain line run every half hour between midnight and six in the morning. What is the probability that a man entering the station at random time during this period will have to wait at least twenty minutes.

e) Define regression of bivariate data (X,Y).

f) Give the statement of Chebychev's ineaquality.

3. Write short answer for each of the following questions: $3 \times 5 = 15$

a) If a random variable has the following probability mass function

X	0	1	2	3	4
$P(X=x)$	k	$\frac{k}{2}$	k	$2k - \frac{1}{5}$	$\frac{9}{20}$

Find $(X \le 1), P(X \le 4), and\ P(X > 1)$

b) A manufacturer of cotter pins knows that 5% of his product is defective. If he sells cotter pins in boxes of 100 and guarantees that not more than 10 pins will be defective, what is the approximate probability that a box will fail to meet the guaranteed quality?

c) Draw the Scatter diagrams which predict the three correlations: positive correlation, negative correlation and zero correlation.

d) If the relation between variables X, Y, U, V are $2X + 3Y = 4, 3U + 4V = 5$ and the regression co-efficient is 4, find the regression co-efficient of y on U.

e) If a random variable X_r $(r = 1,2, \dots n)$ assumes the values r and $-r$ only and all X_r's are independent, show that the law of large numbers cannot be applied here.

4. **Write short answer for each of the following questions:** $4 \times 5 = 20$

a) From an urn containing 3 red and 2 black balls, a man is to draw 2 balls at random with replacement. If he is to receive Rs. 20 for each red ball draws and Rs. 10 for each black one. Find its expectation.

b) Define the following: r^{th} raw (or non-central) moments of a random variable X, r^{th} mo ment of X about a point 'a' and r^{th} central moment of X. Write down the expression for the central moments in terms of row moments.

c) There are 40 students in a Mathematics (Hons) class in a particular college. If the habit of borrowing books from college library of a student is 2/5 find the minimum number of copies of a book referred in a class, to be kept in the library so to meet more than 90% demand of the student.

d) If $X \sim B(n,p)$ and Y has negative binomial distribution with parameters r and p, prove that $F_X(r-1) = 1 - F_Y(n-r)$.

e) For geometric distribution $P(x) = 2^{-x}; x = 1,2,3,\ldots$ prove that Chebychev's inequality gives $P[|X-2| \leq 2] > \frac{1}{2}$,while the actual probability is $\frac{15}{16}$.

5. **Answer the following questions:** $6 \times 2 = 12$

a) Let X be a continuous random variable with p.d.f

$$f(x) = \begin{cases} ax, 0 \leq x < 1 \\ a, 1 \leq x < 2 \\ -ax + 3a, 2 \leq x < 3 \\ 0, elswhere \end{cases}$$

i) Determine the constant 'a'

ii) Evaluate $P(X \leq 1.5)$

Or

" A random variable X can have all or some moments, but M.G.T does not exist except perhaps at one point" Verify the given statement with an example.

b) Let X denote the sum of the numbers on two fair dice. What is the expectation of X?

Or

A and B are two candidates seeking in I.T.T. The probability that A is selected is 0.5 and the probability that A and B are selected is at most 0.3. Is it possible that probability of B getting selected is 0.9?

6. Write the answer of the following questions: $6 \times 2 = 12$

a) Two random variables X and Y have the following joint probability density function:

$$f(x,y) = \begin{cases} 2-x-y; & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0, & otherwise \end{cases}.$$

Find:

i) Marginal probability density functions of X and Y:

ii) Conditional density functions.

iii) Var(X) and Var(Y).

Or

If two coins are tossed one by one find the probability of each $P[X=x, Y=y]$ where X= number of heads appear in the two throws and Y=0, when tail is absent in the first throw; and Y=1, when tail is present in the first throw. Develop the joint distribution table and identify row or column verifies total probability is unity.

b) Define Beta variate of second kind. Also, find its mean and variance.

Or

Show that Normal Distribution as a limiting form of Binomial Distribution.

7. Write the answer of the following: $6 \times 2 = 12$

a) If two random variables X and Y are independent with $E(X) = \mu_1,\ E(Y) = \mu_2,\ Var(X) = {\sigma_1}^2, Var(Y) = {\sigma_2}^2$, then show that $Var(XY) = {\sigma_1}^2{\sigma_2}^2 + {\mu_1}^2{\sigma_2}^2 + {\mu_2}^2{\sigma_1}^2$ Also show that $\rho_1 : \rho_2 = \sigma_1\mu_2 : \sigma_2\mu_1$, where the correlation co-efficient between X and XY is ρ_1 and between Y and XY is ρ_2.

Or

If three uncorrelated variables x_1, x_2, x_3 have the same standard deviations, find the correlation between $x_1 + x_2$ and $x_2 + x_3$.

b) Obtain the lines of regression from the following data:

x	3	5	6	9	11
y	5	4	3	7	8

Also, find the estimated value of x for y = 7 and the estimated value of y for x = 6.

Or

Let $\{X_k\}$ be a sequence of n independent variables with $P\{X_k = \pm k\} = \frac{1}{2}x^{-\lambda}$ and $P\{X_k = 0\} = 1 - k^{-\lambda}$ where $\lambda \ge 0$. Check whether C.L.T holds or not.

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