DHANAMANJURI UNIVERSITY

Examination- 2025 (June)

Four-year course B.A/B.Sc. 6th Semester (NEP)

Name of Programme	:	**B.A. / B.Sc. Mathematics**
Paper Type	:	**CORE (Theory)**
Paper Code	:	**CMA-318**
Paper Title	:	**Probability Theory and Statistics**
Full Marks	:	**80**
Pass Marks	:	**32**

Duration: 3 Hours

The figures in the margin indicate full marks for the questions.

Answer all the questions

1. **Choose and rewrite the correct answer for each of the following:** **1×3=3**

   a) If $P(x_i)$ represents the probability function of a random variable X at the mass points x_i then which of the following gives the corresponding distribution function?

   i) $\sum_{x_i} P(x_i)$

   ii) $\sum_{x_i \leq x} P(x_i)$

   iii) $\sum_{i=1} P(x_i)$

   iv) $\sum_{i \leq x} P(x_i)$

   b) The variance of Geometric distribution $P(x) = q^x\, p, x = 0,1,2, \ldots$ is given by

   i) $\frac{p}{q^2}$ ii) $\frac{q^2}{p}$ iii) $\frac{q}{p^2}$ iv) $\frac{p^2}{q}$

   c) In which of the following interval correlation coefficient 'r' does lei?

   i) $-1 \leq r \leq 1$

   ii) $-\infty < r \leq \infty$

   iii) $0 \leq r \leq 1$

   iv) $-1 \leq r \leq 0$

2. **Write very short answer for each of the following questions:** **1×6=6**

   a) What is the value of *E(X)* when *X* is a constant 'c'?

b) Let X be a random variable with probability function

$$P(X = \pm 2^x) = \begin{cases} \frac{e^{-1}}{x!}, x = 0,1.2, \ldots \\ 0, otherwise \end{cases}$$

Find the value of M.G.F. of X.

c) Write down the marginal probability density function of bivariate random variable (X, Y).

d) Define covariance of bivariate random variables X and Y.

e) The mean of the Binomial distribution is 5. Show that the variance cannot be equal to 6.

f) Define Gamma distribution of continuous random variable X.

3. Write short answer for each of the following questions: $3\times5=15$

a) Two urns contain 3 white and 5 black balls and 4 white and 3 black balls respectively. One ball is drawn at random from each urn. Find the probability distribution of the random variable associated with black balls.

b) If $Z = a + bY$ and r is the correlation coefficient between X an Y, show that

$\sigma_Z{}^2 = a^2\sigma_X{}^2 + b^2\sigma_Y{}^2 + 2abr\,\sigma_X\sigma_Y$ where σ_X and σ_Y are the standard deviations of X and Y; r the correlation coefficient.

c) For Negative Binomial distribution $f(x; r, p)$, prove that

$$f(x+1; r, p) = \frac{x+r}{x+1} q . f(x; r, p)..$$

d) If a die is thrown repeatedly until Six appears, find the probability that it will appear within 3 trials

e) For a Binomial distribution with mean 5 and standard deviation 2, find the mode.

4. Write the answer of the following questions: $4\times5=20$

a) Define cumulative distribution function of a random variable X. If F is the cumulative distribution function of the random variable X and if $a < b$, prove that

$$P(a < X \leq b) = F(b) - F(a).$$

b) If $f(x) = \begin{cases} = \frac{1}{2}, -1 < x < 1 \\ = 0, \ otherwise \end{cases}$

is the density function of a continuous random variable X, find its characteristic function.

c) "A random variable may not have moments although its M.G.F. exists." Justify the given statement with an example.

d) Define correlation coefficient between the bivariate X and Y. Prove that covariance is independent of change of scales.

e) Examine whether C.L.T. holds or not if $P(X_k = \pm 2^k) = \frac{1}{2}$.

5. Answer following questions: **6×2=12**

a) A random variable X has the following probability function values of X:

x	0	1	2	3	4	5	6	7
$P(x)$	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

i) Find the value of k.

ii) Evaluate $P(X < 6)$ and $P(0 < X < 5)$

iii) Determine the distribution function of X.

Or

The random variable X has probability function of the following form, where k is some number,

$P(x) = k \; if \; x = 0;\; P(x) = 2k \; if \; x = 1;\; P(x) = 3k \; if \; x = 2 \; and \; P(x) = 0, otherwise$

i) Determine the value of k,

ii) What is the smallest value of k for which

$P(X \le k) = \frac{1}{2}$? and

iii) Determine the distribution function of X.

b) What does transformation of random variables deal in Probability Theory?

Let X be a continuous random variable with probability density function $f_X(x)$. Let $y = g(x)$ be strictly monotonic (increasing or decreasing) function of x. Assume that $g(x)$ is differentiable for all x. Prove that probability function of the random variable Y is given by

$h_Y(x) = f_X(x)\left|\frac{dx}{dy}\right|$ where x is expressed in terms of y.

Or

A special dice with (n+1) faces is marked on its faces the number 0, $\frac{1}{n}, \frac{2}{n}, \frac{3}{n}, \ldots, \frac{n}{n}$. The dice is unbiased. For the distribution of the random variable corresponding to the number n the uppermost face. Find (a) the expected value and (b) the standard deviation.

6. Write the answer of the following questions: **6×2=12**

a) Define Binomial distribution. Derive Poisson distribution as the limiting case of Binomial distribution.

Or

Define Beta variate of first kind and find its mean and variance.

b) The daily consumption of milk in a city, in excess of 20,000 litres, is approximately distributed as a Gamma variate with parameters $a = \frac{1}{10,000}$ and $\lambda = 2$. The city has a daily stock of 30,000 litres. What is the probability that the stock is insufficient on a particular day?

Or

Define a joint probability mass function. A two-dimensional random variable (X, Y) have a joint probability mass function: $P(x, y) = \frac{1}{27}(2x + y)$, where x and y can assume only the integer values 0,1 and 2. Find the conditional distribution of Y for $X = x$.

7. Write the answer of any two questions from the following: **6×2=12**

a) State and prove Chebyshev's inequality.

b) State and prove the De- Moivre-Laplace theorem on CLT.

c) Let (X, Y) be a two dimensional random variable with $E(X) = \bar{X}, E(Y) = \bar{Y}, V(X) = \sigma_X{}^2$ $V(Y) = \sigma_Y{}^2$ and let $r = r(X, Y)$ be the correlation coefficient between X and Y. If the regression of Y on X is linear, prove that $E(Y|X) = \bar{Y} + r.\frac{\sigma_Y}{\sigma_X}(X - \bar{X})$.

Similarly, if the regression of X on Y is linear, prove that $E(X|Y) = \bar{X} + r.\frac{\sigma_X}{\sigma_Y}(Y - \bar{Y})$.

*****