DHANAMANJURI UNIVERSITY

Examination- 2026 (June)

Name of Programme : **B.A./B.Sc. Mathematics**
Semester : **4th**
Paper Type : **Core**
Paper Code : **CMA-212**
Paper Title : **Numerical Analysis**
Full Marks : **80**
Pass Marks : **32**
Duration: 3 Hours

The figures in the margin indicate full marks for the questions.

Answers the following questions:

1. **Choose and rewrite the correct answer for each of the following questions:** $1 \times 3 = 3$

a) If $f(x)$ is a polynomial of degree n+1, then
   i) $\Delta^n f(x)$ is necessarily zero.
   ii) $\Delta^{n-1} f(x)$ is necessarily a constant number.
   iii) $\Delta^{n+1} f(x)$ is necessarily a constant number.
   iv) $\Delta^{n+2} f(x)$ is not zero.

b) Pick out the wrong statement from the followings:
   i) We use Newton's forward interpolating polynomial to evaluate the derivative of $f(x)$ at a point near the beginning of the tabular data.
   ii) We use Newton's backward interpolating polynomial to evaluate the derivative of $f(x)$ at a point near the end of the tabular data.
   iii) When using Newton's divided difference formula, if the third-order difference is constant, the function being approximated is a cubic polynomial.
   iv) When using Newton's divided difference formula, if the third-order difference is constant, the function being approximated is a quartic polynomial.

c) The equation $x^3 - x - 4$ has
   i) a real root in [0,1]
   ii) a real root in [1,2]
   iii) no real root in [1,2]
   iv) a real root in [2,3]

2. Write very short answer for each of the following questions: $1 \times 6 = 6$

a) Evaluate $\Delta(e^{2x})$.
b) What is meant by interpolation?
c) Write the relation between Δ (delta) and E (shift) operators.
d) Write the general quadrature formula.
e) What is meant by the order two in the Runge-Kutta method of order two?
f) What is a diagonally dominant Matrix?

3. Write short answer for each of the following: $3 \times 5 = 15$

a) Evaluate $\left(\frac{\Delta^2}{E}\right)x^3$
b) Construct backward difference table from the give tabular data:

x	1	3	5	7	9
$y = f(x)$	8	12	21	36	62

c) Differentiate the Newton divided difference interpolation formula.
d) Derive the general formula of Euler method for evaluating $y = f(x)$ at a particular point$x$ for the initial value problem: $\frac{dy}{dx} = f(x, y)\,; y(x_0) = y_0$.
e) Perform Regula-Falsa method up to 3rd approximation to find a real root of the equation $x^3 - 3x - 5 = 0$.

4. Write short answer for each of the following: $4 \times 5 = 20$

a) Represent $f(x) = 2x^3 - 3x^2 + 3x - 10$ in factorial notation, the interval of difference being unity and hence evaluate $\Delta^4 f(x)$.
b) Find a Lagrange's interpolating polynomial from the given tabular data:

x	-1	0	2	5
$y = f(x)$	9	5	3	15

c) Evaluate $\int_0^4 e^x\,dx$ by using Simpson's $\frac{1}{3}$ rule.

d) Suppose that $\phi(x)$ is a continuous function in the interval $[a, b]$ that contains the roots of $f(x) = 0$ which is rewritten as $x = \phi(x)$. If $|\phi'(x)| \leq l < 1$ in the interval $[a, b]$, then prove that for any choice of $x_0 \in [a, b]$ the sequence $\{x_n\}$ determined from the fixed point iteration scheme $x_{n+1} = \phi(x_{(n)})$; n = 0,1,2,.. converge to the root.

e) Solve the system of equations $2x - y + 3z = 9, x + y + z = 6, x - y + z = 2$ by using Gauss-Jordan elimination method and verify the result.

5. Answer any one from the following questions: $6 \times 2 = 12$

a) Prove that $\Delta^n x^{(n)} = n!\, h^n$ and hence show that $\Delta^{n+1} x^{(n)} = 0$, where $x^{(n)}$ denotes the factorial notation.

b) Evaluate $f(1.2)$ and $f(1.6)$ based on the following tabular data using Newton-Gregory forward interpolation formula.

x	0	1	2	3	4
$y = f(x)$	1	1.5	2.2	3.1	4.3

c) Construct Newton divided difference table using the following tabular data and hence evaluate $f(2)$ and $f(8)$.

x	4	5	7	10	11	13
$y = f(x)$	48	100	294	900	1210	2028

6. Answer any two of the following questions: $6 \times 2 = 12$

a) From the following tabular data, calculate $\left(\frac{dy}{dx}\right)_{x=1.5}$ and $\left(\frac{dy}{dx}\right)_{x=1}$

x	0	1	2	3	4	5
$y = f(x)$	1	5	31	121	34	781

b) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Simpson's $\frac{3}{8}$ rule and hence estimate the value of π.

c) Evaluate $y(1.1)$ up to 3rd approximation using Runge-Kutta method of order 4 for the initial value problem $\frac{dy}{dx} = x^2 + y^2$; $y(1) = 0$.

7. Answer **any 2 (two)** of the following questions: $6 \times 2 = 12$

a) Locate the roots of $x^3 - 9x + 1 = 0$ and evaluate the smallest one by using bisection method up to sixth approximations. (Correct to two decimal places).

b) Derive the Newton-Raphson method for approximating the root of the equation $f(x) = 0$and hence, using the method so derived approximate a positive root of $x^2 + 2x - 2 = 0$ up to 3rd approximation.

c) Solve the following system of equations $6x + 2y - z = 4$, $x + 5y + z = 3$, $2x + y + 4z = 27$ by using Jacobi method performing up to 5th approximations. Use $(x, y, z) = (0,0,0)$ as initial value.

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