DHANAMANJURI UNIVERSITY

Examination- 2026 (June)

Name of Programme	:	**B.A./B.Sc. Mathematics**
Semester	:	**4th**
Paper Type	:	**Core**
Paper Code	:	**CMA-211**
Paper Title	:	**Mechanics**

Full Marks : 80

Pass Marks : 32 **Duration: 3 Hours**

The figures in the margin indicate full marks for the questions.

Answers the following questions:

1. Choose and rewrite the correct answer for each of the following questions: $1 \times 3 = 3$

   a) The components of velocity of a particle describing a plane curve $r = f(\theta)$ at a point (r, θ) along and perpendicular to radius vector respectively are

   i) $\dot{r}$ and $r\dot{\theta}$ ii) $\dot{r}$ and $r^2\dot{\theta}$

   iii) $\dot{r}$ and $\frac{1}{r}\frac{d}{dt}(r^2\dot{\theta})$ iv) $\dot{r}\theta$ and $\frac{d}{dt}(r^2\dot{\theta})$

   b) A heavy uniform rod 15 inches long is suspended from a fixed point by strings fastened to its ends, their lengths being 9 and 12 inches. If θ be the angle at which the rod is inclined to the vertical, then $25 \sin\theta$ is equal to

   i) 23 ii) 24

   iii) 25 iv) 26

   c) A thin uniform rod of length l is bent into the form of a circular arc whose radius a is large compared to l. Then the displacement of the C.G. is

   i) $\frac{l^2}{a}$ ii) $\frac{24l}{a}$

   iii) $\frac{l^2}{24a}$ iv) $\frac{l}{a^2}$

2. Write very short answer for each of the following questions: $1 \times 6 = 6$

   a) Define terminal velocity for a body falling in a resisting medium under gravity.

b) Write the significance of $h = r^2\dot{\theta}$ for central orbit.

c) Define moment of couple.

d) A smooth sphere of radius r and weight W, is supported in contact with a smooth vertical wall by a strin o length l fastened to a point on it surface and the other being attached to a point in the wall. Show that the tension in the string is $\frac{W(l+r)}{\sqrt{l^2+2lr}}$.

e) Define coefficient of friction.

f) Find the C.G. of the area included between the curve $Y^2(2a-x) = x^3$ and its asymptotes.

3. Answer **any 5 (five)** of the following questions: $3 \times 5 = 15$

a) An insect crawls at a constant rate U along the spoke of a cart wheel of radius R. The cart is moving with a velocity V. Find the accelerations along and perpendicular to the spoke.

b) A particle describes a plane curve with a uniform velocity V for which S and ψ vanish simultaneously. If the acceleration at (S,ψ) be $\frac{CV^2}{C^2+S^2}$, C being a constant, show that the intrinsic equation to the path is $S = C\tan\psi$.

c) $ABCD$ is a rectangle such that $AB = CD = a$ and $BC = DA = b$. Forces P act along AD and CB and forces Q act along AB and CD. Prove that the perpendicular distance between the resultant of P, Q at C is $\frac{Qb-Pa}{\sqrt{P^2+Q^2}}$.

d) If a line CD be drawn through the vertex C of a ΔABC meeting the opposite side AB in D and dividing it into two parts m and n and the angle C into two parts α and β and if $\angle CDB = \theta$, then prove that (i) $(m+n)\cot\theta = m\cot\alpha - n\ cot\beta$ and (ii) $(m+n)\cot\theta = n\cot A - m\cot B$.

e) A uniform ladder is in equilibrium with one end resting on the ground and the other against a vertical wall, if the ground and wall be both rough, the coefficients of friction being μ and μ' rerspectively and if the ladder be on the point of slipping at both ends, then show that the inclination of the ladder to the hoirizon is $\theta = \tan^{-1}\left(\frac{1-\mu\mu'}{2\mu}\right)$.

f) A triangular lamina ABC hangs at rest, one of the angles A being supported at a fixed point. Find the angle which the lower side makes with the horizon.

g) Find the position of the centroid of the area of the cardioid $r = a(1 + \cos\theta)$.

4. Answer **any 5 (five)** from the following questions: $4 \times 5 = 20$

a) A particle of mass m is projected vertically upwards under gravity in a medium, the resistance of which is mk times the velocity. Show that the greatest height attained by the particle is $\frac{V_0^2}{g}[\lambda - \log(1+\lambda)]$ if V_0 is the terminal velocity and λV_0 is the initial velocity of projection.

b) The length of a seconds pendulum be lengthened by $\frac{1}{100}$ of its original length. How many seconds will it lose per day?

c) A uniform square lamina rests in equilibrium under gravity in a vertical plane with two of its sides in contact with smooth pegs in the same horizontal line at a distance c apart. Show that the angle θ made by a side of the square with the horizontal in a nonsymmetrical position of equilibrium is given by $c(\sin\theta + \cos\theta) = a$, where $2a$ is the length of a side of the square.

d) Prove that for any two couples whose moments are equal and opposite act in the same plane upon a rigid body are balance one another.

* e) Find the least force required to pull a body and down on a rough inclined plane.

f) A uniform ladder of length $4a$ rests at an angle α to the horizontal against a smooth horizontal rail at a height h from the ground. If λ be the angle of friction between the ground and the ladder, show that a manof twice the weight of the ladder may ascend a distance $3h \sin\lambda \ \text{cosec}\,(\alpha + \lambda)\ \text{cosec}\, 2\alpha - a$.

g) Fins the C.G. of the area bounded by the axis of y and the cycloid $x = a(\theta + \sin\theta), y = a(1 - \cos\theta)$.

5. Answer **any 6 (six)** from the following questions: $6 \times 6 = 36$

a) Prove that the acceleration $\vec{a}$ of a particle describing a plane curve is $\vec{a} = a_t\hat{e_t} + a_n\widehat{e_n}$ if $a_t = \frac{dv}{dt}$, $a_n = \frac{v^2}{\rho}$, $\rho = \frac{ds}{d\psi}$.

b) A particle moves in a plane curve under a central force $\vec{F}$ which is always directed towards or away from a fixed point in the plane of the forces. Prove with usual notation $\frac{d^2u}{d\theta^2} + u = \frac{F}{h^2U^2}$, $u = \frac{1}{r}$ and $h = r^2\theta$.

c) A spherical raindrop falling freely under gravity receives in each instant an increase of volume equal to λ-times its surface area. Find velocity and distance fallen at any time t. Assume that the raindrop starts from rest.

d) Equal weights P and P are attached to the ends Q and R of two strings ACQ and BCR passing over a smooth peg C and attached to the ends A and B of a heavy beam AB of weight W, whose C.G. is at distances a metre from A and b metre from B, show that $\overline{AB}$ is inclined to the horizon at an angle $\tan^{-1}\left\{\frac{a-b}{a+b}\tan\left(\sin^{-1}\frac{W}{2P}\right)\right\}$.

e) Forces P, Q, R, S are acting along the sides AB, BC, CD and DA of a cyclic quadrilateral $ABCD$. AB is the diameter of the circumscribed circle. If the forces are in equilibrium, then prove that $R^2 = P^2 + Q^2 + S^2 + \frac{2PQS}{R}$.

f) A uniform ladder of length $2a$ and weight w rests with one end against a rough vertical wall and the other upon an equally rough horizontal floor. Show that a man whose weight is P can never get near to the top of the ladder than $\frac{w\cot 2\lambda + P\cot\lambda - (w+P)\tan\theta}{P}a\sin 2\lambda$, where θ is the inclination of the rod to the horizontal and λ is the angle of friction.

g) Find the C.G. of the area of the loop of the curve $r = a\cos 3\theta$ which contains the initial line.

h) Find the centroid of the volume formed by the revolution of the curve $r = a(1+\cos\theta)$ about the x-axis.

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