DHANAMANJURI UNIVERSITY

Examination- 2025 (June)

Four-year course B.A./B.Sc. 4^{th} Semester (NEP)

Name of Programme : B.A./B.Sc. Mathematics
Paper Type : CORE (Theory)
Paper Code : CMA-211
Paper Title : Mechanics
Full Marks : 80
Pass Marks : 32 **Duration: 3 Hours**

The figures in the margin indicate full marks for the questions.
Answer all the questions:

1. Choose and rewrite the correct answer for each ofthe following:

$1 \times 3 = 3$

i) A particle describes a circle of radius R with a uniform speed. If ω be the angular velocity of rotation, then which one of the following is true?

a) The acceleration at any point of the path is $R\omega^2$ and is along the tangent.

b) The acceleration at any point of the path is $R\omega^2$ and is along the normal towards the centre.

c) The acceleration does not exist.

d) The acceleration exists and acts along the direction bisecting the angle between the tangential and normal directions.

ii) $ABCDEF$ is a regular hexagon of side a. Forces $P, 2P, 3P, 2P, 5P, 6P$ act along AB, BC, DC, ED, EF and AF respectively, then the moment of the couple formed is

a) $-2Pa\sqrt{3}$

b)$-3Pa\sqrt{3}$

c) $-3P\sqrt{3}$

d) $-2a\sqrt{3}$

iii) A uniform wire 24 inches long is bent into the shape of a triangle, the sides being 3:4:5. Particles of weights p,q,r are placed at the angular points and it is found that the centre of gravity is unchanged, the p:q:r is equal to

a) 3:4:5

b) 5:4:3

c) 7:8:9

d) 9:8:7

2. Write very short answers for each of the following questions: $1 \times 6 = 6$

i) A bomb shell explodes in such a manner that its fragments fly off with a velocity V in all directions. Find the area of the maximum circle within which all the fragments scatter.

ii) Define terminal velocity for a particle falling under the action of gravity.

iii) How is the algebraic sum of the moments of the forces forming a couple about any point in their plane a non-zero constant and equal to the moment of the couple?

iv) What is the condition that three forces acting on a rigid body must satisfy when the body is in equilibrium?

v) Find the least force required to pull a body on a rough horizontal plane.

vi) Define centre of gravity.

3. Answer any five from the following questions: $3 \times 5 = 15$

i) An insect crawls at a constant rate u along a spoke of a cart wheel of radius R, the cart is moving with a velocity V. Find the radical acceleration of the insect.

ii) Find the intrinsic equation to a curve such that when a particle moves on it with a constant tangential acceleration, the magnitude velocity and the normal acceleration bears a constant ratio.

iii) If three coplanar forces acting on a rigid body be in equilibrium, prove that they must either meet at a point or parallel to one another.

iv) The algebraic sum of the moments of a system of coplanar forces about the points (1,0), (0,2) and (2,3) referred to rectangular axes are G_1, G_2 and G_3 respectively. Find the tangent of the angle which the direction of the resultant force makes with the axis of x .

v) Two rough particles connected by a light string rest on an inclined plane. If their weights and coefficients of friction are W_1, W_2 and μ_1, μ_2 respectively, show that the greatest inclination of the plane for equilibrium is

$$\tan^{-1}\left(\frac{\mu_1 W_1 + \mu_2 W_2}{W_1 + W_2}\right)$$

vi) A uniform ladder of length 70 feet rests against a vertical wall with which it makes an angle of $45°$; the coefficient of friction between the ladder and the wall is $\frac{1}{3}$ and that between the ladder and the floor $\frac{1}{2}$. If a man whose weight is one half that of the ladder ascends it, how high will he be when the ladder slip?

vii) Perpendiculars are drawn from the angular points A, B, C of a triangle to the opposite sides a, b, c and another triangle is formed by joining the feet of these perpendiculars. If x, y, z be the distances of the centre of gravity of this triangle from the sides a, b, c, prove that

$$\frac{x}{a^2\cos(B-C)} = \frac{y}{b^2\cos(C-A)} = \frac{z}{c^2\cos(A-B)}$$

4. Answer any five from the following questions: $4 \times 5 = 20$

i) What is meant by a seconds pendulum? If a seconds pendulum be lengthened by $\frac{1}{100}$th of its original length, how many seconds will it lose in a day?

ii) A particle of unit mass is projected under gravity in a medium whose resistance equals k times the velocity with a velocity u at an angle of elevation α to the horizon. If $\dot{x}$ and $\dot{y}$ are the components of the velocity at a point $P(x, y)$, then show that

$$\dot{x} = u\cos\alpha \cdot e^{-kt} \quad \text{and} \quad g + \dot{y} = (g + u\sin\alpha) \cdot e^{-kt}$$

iii) Establish the equation

$$\frac{d}{dt}(mv) = F + u\frac{dm}{dt}$$

where the symbols have their usual meanings.

iv) Prove that any number of coplanar couples acting on a body is equivalent to a single couple whose moment is equal to the algebraic sum of the moments of the couples.

v) A heavy uniform rod of length L rest with one end against a smooth vertical wall, the other end being tied to a point of the wall by a string of length l. Prove that the rod may remain in equilibrium at an angle ϕ to the wall is

$$\cos^2\phi = \frac{l^2 - L^2}{3L^2}$$

vi) A straight uniform beam of length $2h$ rest in limiting equilibrium in contact with a rough vertical wall of height h, with one end on a rough horizontal plane and with the other end projecting beyond the wall. If both the wall and the plane be equally rough, then prove that λ, the angle of friction is given by $\sin 2\lambda = \sin\alpha \sin 2\alpha$, where α is the inclination of the beam to the horizon.

vii) Find the centre of gravity of the arc of an asteroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ lying in the first quadrant.

5. Answer any six from the following questions: $6 \times 6 = 36$

i) Establish the equation

$$\frac{d^2u}{d\theta^2} + u = \frac{F}{h^2u^2}, \quad u = \frac{1}{r}, \quad r \neq 0$$

ii) A particle falls from rest under the action of gravity in a medium whose resistance is k times (velocity)^2. If V and x be the velocity acquired and height fallen in the time t, then prove that

a) $V = v_0 \tanh\left(\frac{gt}{v_0}\right)$

b) $x = \frac{v_0^2}{g} \log\left(\cosh\left(\frac{gt}{v_0}\right)\right)$

if v_0 is the terminal velocity.

iii) A particle of mass M is at rest and begins to move under the action of a constant force F. It encounters a stream of fine dust moving with a velocity V which deposits matter at a constant rate ρ. Prove that its mass is m when it has travelled

$$\frac{k}{\rho^2}\left[m - M\left(1 + \log\frac{m}{M}\right)\right], \quad k = F - \rho V$$

iv) If two couples whose moments are equal and opposite act in the same plane upon a rigid body, prove that they balance one another.

v) Prove that a system of forces acting in one plane at different points of a rigid body can be reduced to a single force R through any arbitrary point and a couple, whose moment is equal to the sum of the moments of the given forces about this point.

vi) Find the least force required to pull a body up or down a rough inclined plane.

vii) Two equal uniform ladders are joined at one end and stand with the other ends on a rough horizontal plane. A man whose weight is equal to that of the ladders ascends one of them. Prove that the other will slip first.

Supposing that it slips when he has ascended a distance x, prove that the coefficient of friction is

$$\frac{a+x}{2a+x}\tan\alpha$$

where a is the length of each ladder and α the angle which each makes with the vertical.

viii) Find the centre of gravity of the arc of the parabola $y^2 = 4ax$ extending from the origin (vertex) to the extremity of the latus rectum.

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