DHANAMANJURI UNIVERSITY

Examination- 2026 (June)

Name of Programme	:	**B.A./B.Sc. Mathematics**
Semester	:	**4th**
Paper Type	:	**Core**
Paper Code	:	**CMA-210**
Paper Title	:	**Riemann Integration and Series of Functions**

Full Marks : 80

Pass Marks : 32 **Duration: 3 Hours**

The figures in the margin indicate full marks for the questions.

Answers the following questions:

1. Choose and rewrite the correct answer for each of the following questions: $1 \times 3 = 3$

   a) The function $f(x)=[x]$, where $[x]$ denotes the greatest integer not greater than x, is integrable on $[0, 3]$. The number of finite point of discontinuity for the function f is/are

   i) 1 ii) 2

   iii) 3 iv) 4

   b) The improper integral $\int_a^\infty \frac{1}{x^p} dx,\ a > 0$, converges if and only if

   i) $p>0$ ii) $p<0$

   iii) $p<1$ iv) $p>1$

   c) The radius of convergence of power series $\sum a_n x^n$ is R. Then for any positive integer m, the radius of convergence of power series $\sum a_n x^{mn}$ is

   i) R ii) $R^{\frac{1}{n}}$

   iii) $R^{\frac{1}{m}}$ iv) $R^{\frac{1}{mn}}$

2. Write very short answer for each of the following questions: $1 \times 6 = 6$

a) If $f(x) = 3x - 2$ and $P = \{0, 3, 4, 6\}$ is a partition of $[0, 6]$, then find the value of $W(P, f)$.

b) State fundamental theorem of integral calculus.

c) Examine the convergence of improper integral $\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx$.

d) State Dirichlet's test for improper integral.

e) When is a series of functions $\sum f_n$ said to be uniformly convergence in an interval $[a, b]$?

f) State Cauchy-Hadamard theorem for power series.

3. Write short answer for each of the following: $3 \times 5 = 15$

a) Prove that if P^* is a refinement of a partition P, for a bounded function f on $[a, b]$, then prove that $U(P^*, f) \leq U(P, f)$.

b) Prove that every continuous function on $[a, b]$ is integrable on $[a, b]$.

c) Test the convergence of improper integral $\int_{-\infty}^{0} \cosh x dx$.

d) Show that improper integral $\int_{0}^{\infty} \frac{3}{(x+6)\sqrt{x}} dx$ is convergent with value $\frac{\sqrt{6}}{2}\pi$.

e) Show that the series $\sum \frac{\cos n\theta}{n^p}$ is uniformly and absolutely convergent for all real values of $\theta, p > 1$.

4. Write short answer for each of the following: $4 \times 5 = 20$

a) State and prove Darboux's theorem for Upper Riemann Sum.

b) Prove that if f is bounded and integrable over $[a, b]$, then there exists a number μ lying between the bounds of f in $[a, b]$ such that $\int_a^b f(x)dx = \mu(b-a)$.

c) Show that the improper integral $\int_0^{\pi/2}\left(\frac{\sin^m x}{x^n}\right)dx$ exists if and only if $n < m+1$.

d) Test the uniform convergence of the sequence of functions $\{f_n\}$, where $f_n(x) = nxe^{-nx^2}, x > 0$.

e) Determine the radius of convergence and the exact interval of convergence of the power series $\sum \frac{nx^n}{(n+1)^2}$.

5. Answer **any 2 (two)** of the following questions: $6 \times 2 = 12$

a) State a necessary and sufficient condition for integrability of a bounded function f on an interval $[a, b]$ and prove the same.

b) If f_1 and f_2 are two bounded and integrable functions on $[a, b]$, then prove that their product $f_1 f_2$ is also bounded and integrable on $[a, b]$.

c) Prove that a bounded function f is integrable on $[a, b]$ if the set of its points of discontinuity has only a finite number of limit points.

6. Answer **any 2 (two)** of the following questions: $6 \times 2 = 12$

a) Show that $\int_0^1 x^{m-1}(1-x)^{n-1}dx$ converges if and only if $m, n > 0$.

b) Test the convergence of the improper integral $\int_{-1}^{1}\frac{dx}{(2-x)\sqrt{1-x^2}}$.

c) State and prove Frullani's theorem for improper integral.

7. Answer **any 2 (two)** of the following questions: $6 \times 2 = 12$

a) State Cauchy's Criterion for uniform convergence for sequence of functions. Also prove the same.

b) Show that the sequence $\{f_n\}$, where $f_n(x) = \frac{x}{1+nx^2}$ converges uniformly to a function f on [0, 1], and that the equation $f'(x) = \lim_{n\to\infty} f_n'(x)$ is true if $x \neq 0$ and false if $x = 0$. Why so?

c) State and prove Abel's theorem for power series.

*****