DHANAMANJURI UNIVERSITY
DECEMBER 2025

Name of Programme : **B.A./B.Sc. Mathematics**
Semester : **3rd**
Paper Type : **Core**
Paper Code : **CMA-208**
Paper Title : **Partial Differential Equations**
Full Marks : **80**
Pass Marks : **32**
Duration: 3 Hours

The figures in the margin indicate full marks for the questions.

1. Choose and rewrite the correct answer for each of the following questions: 1X3=3

a. The partial differential equation obtained by eliminating the arbitrary constants a and b from the equation $z = \frac{x^2}{a} + \frac{y^2}{b}$ is

i. $z = px + qy$
ii. $2z = px + qy$
iii. $2z = qx + py$
iv. $z = px - qy$

b. A partial differential equation is said to be of Clairaut form, if it can be written as

i. $z = py + qx + f(x, y)$
ii. $z = px + py + f(a, b)$
iii. $z = px + py + f(p, q)$
iv. $z = ax + by + f(a, b)$

c. The equation $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x\, \partial y} - 8\frac{\partial^2 z}{\partial y^2} = 0$ is a/an _____.

i. Elliptic
ii. Circle
iii. Parabolic
iv. Hyperbolic

2. Write very short answer for each of the following: 1X6=6

a) Form the partial differential equation by eliminating the arbitrary constants a and b from $z = ax + by + ab$.

b) Define singular integral of a first order partial differential equation.

c) Write the complete integral of $z = px + qy + p^2 + q^2$.

d) If $s = 2x + 2y$, find q.

e) Write the complementary function of $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 12(x + y)$.

f) Define a linear homogeneous partial differential equation.

3. Write short answer for each of the following: 3X5=15

a) Solve by using method of characteristics $2u_x - u_y = 0,\ u(x, 0) = f(x)$.

b) Solve : $yp = 2yx + \log q$.

c) Apply Charpit's method to find the complete integral of $z = px + qy + p^2 + q^2$.

d) Solve : $(D^2 - 6DD' + 9D'^2)z = 6x + 2y$

e) Solve : $xr + p = 9x^2y^2$.

4. Write short answer for each of the following: 4X5=20

a) Form a partial differential equation by eliminating the arbitrary function f from $f(x + y + z,\ x^2 + y^2 - z^2) = 0$. Also state the order of the partial differential equation so obtained.

b) Find the partial differential equation of all surfaces of revolution having z-axis as the axis of the revolution.

c) Find the complete integral of $p^3 + q^3 = 27z$.

d) Find a real function V of x and y, reducing to zero where $y = 0$ and satisfying $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = -4\pi(x^2 + y^2)$.

e) Solve: $(D - D' - 1)(D - D' - 2)z = e^{2x-y} + x$.

5. Answer any two questions: 6X2=12

a) Define partial differential equation. Also give the definitions of all types of partial differential equation with suitable examples.

b) Show that the differential equation of all cones which have their vertices at the origin is $px + qy = z$.

c) Solve by using method of separation of variables $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$, if $u(0, y) = 8e^{-3y}$.

6. Answer any two questions: 6X2=12

a) Find the complete integral, general integral and singular integral of the partial differential equation involving only p and q not x,y,z

b) Solve : $(p^2 + q^2)y = qz$ by using Charpit's method.

c) Find the complete integral of the equation $2p_1x_1x_3 + 3p_2{x_3}^2 + {p_2}^2p_3 = 0$ by using Jacobi's method.

7. Answer any two questions: 6X2=12

a) Solve : $(D^2 - DD' - 2D'^2)z = (y - 1)e^x$.

b) Solve : $x^2r - 3xys + 2y^2t + px + 2qy = x + 2y$.

c) Solve by Monge's method $r - t\cos^2 x + p\tan x = 0$.

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