DHANAMANJURI UNIVERSITY

Examination, 2023 (Dec)

Four year course B.A/B.Sc. 3rd semester

Name of Programme	:	**B.A/B.Sc. Mathematics (Honours)**
Semester	:	**III**
Paper Type	:	**Core-VIII (Theory)**
Paper Code	:	**CMA-208**
Paper Title	:	**Partial Differential Equations**

Full Marks : 80

Pass Marks : 32 **Duration: 3 Hours**

The figures in the margin indicate full marks for the questions.

Answer all questions.

1. **Choose the correct answer from each of the following and rewrite it:** $\mathbf{1 \times 4 = 4}$

   a) The partial differential equation by eliminating arbitrary function from the equation $z = e^{ny} f(x - y)$ is

      i) $p + q = nz$ ii) $q - p = nz$

      iii) $p - q = nz$ iv) $pq = nz$

   b) The singular solution of $z = px + qy + pq$ is

      i) $z = -2 - \log xy$ ii) $z = 2 - \log xy$

      iii) $z = \log xy - 2$ iv) $z = 2 + \log xy$

   c) The complete integral of $f(p, q) = 0$ by Charpit's method is

      i) $z = x + y + c$ ii) $z = ax + y + c$

      iii) $z = x + by + c$ iv) $z = ax + by + c$

   d) The operators $2\frac{\partial^2 u}{\partial t^2} + 4\frac{\partial^2 u}{\partial x\,\partial t} + 3\frac{\partial^2 u}{\partial x^2}$ represent

      i) a circle ii) an elliptic

      iii) Hyperbolic iv) a parabolic

2. **Write very short answer for each of the following questions:** $\mathbf{1 \times 10 = 10}$

   a) Define the order of a partial differential equation.

   b) Form the partial differential equation of $z = f\left(\frac{xy}{z}\right)$ by eliminating arbitrary function.

   c) Write the complete integral of the equation of the form $f(x, p) = F(y, q)$.

   d) Define general integral of the first order partial differential equation.

   e) Write the complete integral of $pq = 1$.

   f) State the form of the partial differential equation $p^3 + q^3 = 27z$.

   g) Write the complete integral of $p = e^q$.

   h) Find the solution of $s = 0$.

   i) Solve $r = \sin(xy)$.

   j) Write Monge's subsidiary equations of $r = a^2 t$.

3. Write short answer for each of the following questions: $\mathbf{3 \times 10 = 30}$

a) Derive the partial differential equation by eliminating arbitrary constants.

b) Find the integral surface of the Cauchy problem $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 1$, which passes through the circle $z = 0, x^2 + y^2 = 1$.

c) Form a partial differential equation by eliminating the function f from $z = y^2 + 2f(\frac{1}{x} + \log y)$.

d) Find the complete integral of $(y - x)(qy - px) = (p - q)^2$.

e) Solve: $z(p^2 - q^2) = x - y$.

f) Solve: $2zx - px^2 - 2qxy + bq = 0$ by Charpit's method.

g) Solve: $t - xq = x^2$.

h) Solve: $\left(D^3 - 4D^2D' + 4DD'^2\right)z = 4\sin(2x + y)$.

i) Solve: $(D - D' - 1)(D - D' - 2)z = e^{2x-y} + x$.

j) Solve: $x^2 \frac{\partial^2 z}{\partial x^2} - y^2 \frac{\partial^2 z}{\partial y^2} - y\frac{\partial z}{\partial y} + x\frac{\partial z}{\partial x} = 0$.

4. Answer any two questions from the following questions: $\mathbf{6 \times 2 = 12}$

a) Derive the formula for partial differential equation by eliminating arbitrary functions.

b) Form a partial differential equation by eliminating a, b, c from $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

c) Show that the differential equation of all cones which have their vertex at the origin is $px + qy = z$. Verify that $yz + zx + xy = 0$ is a surface satisfying the above equation.

5. Answer any two questions from the following questions: $\mathbf{6 \times 2 = 12}$

a) Solve : $(x^2 + y^2)(p^2 + q^2) = 1$.

b) Apply Charpit's method to find the complete integral of $p^2 + q^2 - 2px - 2qy + 2xy = 0$.

c) Describe Jacobi method of solving a general first order partial differential equation $f(x, y, z, p, q) = 0$. Illustrate the same equation $p^2x + q^2y = z$.

6. Answer any two question from the following questions: $\mathbf{6 \times 2 = 12}$

a) Find the surface passing through the parabolas $z = 0, y^2 = 4ax \; and \; z = 1, y^2 = -4ax$ and satisfying the equation $xr + 2p = 0$.

b) Solve : $x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \, \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} - nx\frac{\partial z}{\partial x} - ny\frac{\partial z}{\partial y} + nz = x^2 + y^2$.

c) Solve : $y^2r - 2ys + t = p + 6y$ by Monge's method.

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