# DHANAMANJURI UNIVERSITY

Examination, 2023 (Dec)

Four year course B.A/B.Sc. 3rd semester

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| **Name of Programme** | **:** | **B.A/B.Sc. Mathematics (Honours)** |
| **Semester** | **:** | **III** |
| **Paper Type** | **:** | **Core-VII (Theory)** |
| **Paper Code** | **:** | **CMA-207** |
| **Paper Title** | **:** | **Theory of Real Functions** |

**Full Marks : 80**

**Pass Marks : 32** **Duration: 3 Hours**

***The figures in the margin indicate full marks for the questions.***

***Answer all questions.***

1. Using $\varepsilon - \delta$ definition, show that $\lim_{x \to a} x^2 = a^2$. 4

2. State Squeeze theorem. Using the same, show that $\lim_{x \to 0} x^{\frac{3}{2}} = 0$. 4

3. Prove that every uniformly continuous function on an interval $I$ is continuous on $I$. 4

4. By using $\varepsilon - \delta$ definition, prove that the sine function is continuous on $\mathbb{R}$. 4

5. Let $f: A \to \mathbb{R}$ and let $c$ be a cluster point of $A$. Then, prove that the following statements are equivalent:

   a) $\lim_{x \to c} f(x) = L$

   b) Given any $\varepsilon -$ neighbourhood $V_\varepsilon(L)$ of $L$, there exists a $\delta -$neighbourhood $V_\delta(c)$ of $c$ such that $x \neq c$ is any point in $V_\delta(c) \cap A$, then $f(x)$ belongs to $V_\varepsilon(L)$. 8

   Or

   State Sequential Criterion theorem for limits. Prove the same. 8

6. **Answer any two questions from the following:** $\mathbf{8 \times 2 = 16}$

   a) Let $I = [a, b]$ be closed bounded interval and $f: I \to \mathbb{R}$ be continuous on $I$. Then, prove that $f$ is bounded on $I$.

   b) Let $I = [a, b]$ be a closed bounded interval and let $f: I \to \mathbb{R}$ be continuous on $I$. Then, show that $f$ has an absolute maximum and an absolute minimum on $I$.

   c) Let $I = [a, b]$ and let $f: I \to \mathbb{R}$ be continuous on $I$. If $f(a) < 0 < f(b)$ or if $f(a) > 0 > f(b)$. Then, prove that there exists a number $c \in (a, b)$ such that $f(c) = 0$.

7. **Answer any three questions from the following:** $\mathbf{8 \times 3 = 24}$

   State and prove

   a) Caratheordary's theorem.

   b) Interior Extremum theorem.

   c) Rolle's theorem

   d) Darboux's theorem.

8. **Answer any two questions from the following:** $\mathbf{8 \times 2 = 16}$

   a) State Taylor's theorem in finite form with Lagrange's form of remainder. Prove the same.

   b) Let $I$ be an interval, let $x_0$ be an interior point of $I$, and let $n \geq 2$. Suppose that the derivatives $f', f'', \ldots, f^{(n)}$ exist and are continuous in a neighbourhood of $x_0$ and that $f'(x_0) = \cdots = f^{(n-1)}(x_0) = 0$, but $f^{(n)}(x_0) \neq 0$ and $n$ is even. Then prove that $f$ has a relative maximum or a relative minimum at $x_0$ according as $f^{(n)}(x_0) > 0$ *or* $f^{(n)}(x_0) < 0$.

   c) Let $I$ be an open interval and let $f: A \to \mathbb{R}$ have a second derivative on $I$. Then, prove that $f$ is a convex function on $I$ if and only if $f''(x) \geq 0$ for all $x \in I$.

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