# DHANAMANJURI UNIVERSITY

## Examination- 2026 (June)

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| Name of Programme | : | B.A./ B.Sc. Mathematics |
| Semester | : | 2nd |
| Paper Type | : | Core |
| Paper Code | : | CMA-106 |
| Paper Title | : | Vector Analysis and Solid Geometry |

Full Marks : 80

Pass Marks : 32 Duration: 3 Hours

***The figures in the margin indicate full marks for the questions.***

***Answer all the questions:***

**1. Choose and rewrite the correct answer:** $1 \times 3 = 3$

a) The vector triple product $\vec{a} \times (\vec{b} \times \vec{c})$ is equal to

i) $(\vec{a} \cdot \vec{b})\vec{c} - (\vec{b} \cdot \vec{a})\vec{c}$ ii) $(\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$

iii) $(\vec{a} \cdot \vec{b})\vec{c} - (\vec{a} \cdot \vec{c})\vec{b}$ iv) $(\vec{a} \cdot \vec{c})\vec{b} - (\vec{b} \cdot \vec{c})\vec{a}$

b) The radius of the sphere $x^2 + y^2 + z^2 - 2x + 4y - 6z = 2$ *is*

i) 5 ii) 8

iii) 16 iv) 4

c) The equation $5x^2 - 4y^2 + 11z^2 = 1$ represents

i) an ellipsoid ii) a hyperboloid of one sheet

iii) a hyperboloid of two sheets iv) an elliptic paraboloid

**2. Write very short answer for each of the following:** $1 \times 6 = 6$

a) Find $\vec{a} \times \vec{b}$, where $\vec{a} = \hat{\imath} - \hat{\jmath} + 2\hat{k}$ and $\vec{b} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}$.

b) Find $div\, \vec{r}$, where $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$.

c) Write the equation of the sphere described on the segment joining the points $A(x_1, y_1, z_1), B(x_2, y_2, z_2)$ as diameter.

d) What is the semi vertical angle of a right circular cone having sets of three mutually perpendicular tangent plane?

e) Define Director Sphere of a central conicoid.

f) Write the name of the surface represented by the equation $4x^2 + 9y^2 + 11z^2 = 1$.

3. **Write short answers for each of the following:** $3 \times 5 = 15$

a) If $A = (3x^2 + 6y)\hat{\imath} - 14yz\hat{\jmath} + 20xz^2\hat{k}$, evaluate $\int_C A \cdot dr$, where $C$ is the line from $(0,0,0)$ to $(1,0,0)$ then to $(1,1,0)$ and then to $(1,1,1)$.

b) Find the equation of the sphere on which the circle given by $x + y + z + 3 = 0$ and $x^2 + y^2 + z^2 = 9$ is a great circle.

c) Find the equation of the cone whose vertex is $(\alpha, \beta, \gamma)$ and base $ax^2 + by^2 = 1, z = 0$.

d) Show that $2x^2 + 2y^2 + 7z^2 - 10yz - 10zx + 2x + 2y + 26z - 17 = 0$ represent a cone by using the condition of second degree to represent a cone.

e) Find the points of intersection of the line $\frac{x}{3} = \frac{y+1}{-1} = \frac{z+1}{1}$ with the conicoid $x^2 - 3y^2 + 4z^2 = 1$.

4. **Answer the following questions:** $4 \times 5 = 20$

a) If $f$ is a scalar valued function and $F$ is a vector valued function, show that $\nabla \times (fF) = \nabla f \times F + f(\nabla \times F)$.

b) If $\vec{r} = \text{a} \cos t\, \hat{\imath} + \text{a} \sin t\, \hat{\jmath} + at \tan \alpha\ \hat{k}$, then find the value of $\left|\frac{d\vec{r}}{dt} \times \frac{d^2\vec{r}}{dt^2}\right|$ and $\frac{d\vec{r}}{dt} \cdot \frac{d^2\vec{r}}{dt^2} \times \frac{d^3\vec{r}}{dt^3}$.

c) Show that $(-1, -2, -3)$ is the vertex of the cone. $4x^2 - y^2 + 2z^2 + 2xy - 3yz + 12x - 11y + 6z + 4 = 0$

d) Find the equation of the sphere through the points $(0.0,0)$, $(0,1,-1), (-1,2,0), (1,2,3)$.

e) Find the condition that the plane $lx + my + nz = p$ should touch the paraboloid $ax^2 + by^2 = 2cz$.

**5. Answer any two of the following:** $6 \times 2 = 12$

a) Verify Green's theorem in the plane for $\oint_{\Gamma} x^2dx + xydy$ where $\Gamma$ is a square in the $xy$ –plane given by $x = 0, y = 0, x = a, y = a \ (a > 0)$, described in the positive sense.

b) State and Prove Green's theorem.

c) Verify Gauss Divergence theorem, given that $F = 4xz\hat{\imath} - y^2\hat{\jmath} + yz\hat{k}$ and $S$ is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.

**6. Answer any two of the following:** $6 \times 2 = 12$

a) Prove that the equation of the tangent plane at any point $(\alpha, \beta, \gamma)$ of the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ is $(\alpha + u)x + (\beta + v)y + (\gamma + w)z + (\alpha u + \beta v + \gamma w + d) = 0$

b) Find the condition that the second degree equation $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy + 2ux + 2vy + 2wz + d = 0$, may represent a cone.

c) A sphere of constant radius $k$ passes through the origin and cuts the axes in $A, B, C$. Find the locus of the centroid of the triangle $ABC$.

**7. Answer any two of the following:** $6 \times 2 = 12$

a) Prove that the locus of the foot of the central perpendicular on varying tangent planes of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ is the surface $(x^2 + y^2 + z^2)^2 = a^2x^2 + b^2y^2 + c^2z^2$.

b) Find the equation of the enveloping cone from the point $(\alpha, \beta, \gamma)$ to the conicoid $ax^2 + by^2 + cz^2 = 1$.

c) Find the locus of the chords of the conicoid $ax^2 + by^2 + cz^2 = 1$ which are bisected at a given point $(\alpha, \beta, \gamma)$.

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