DHANAMANJURI UNIVERSITY

Examination- 2025 (June)

Four-year course B.A/B.Sc. 2nd Semester

Name of Programme	:	**B.A. /B. SC. Mathematics**
Paper Type	:	**Core-VI (Theory)**
Paper Code	:	**CMA-106**
Paper Title	:	**Vector Analysis & Solid Geometry**

Full Marks : 80

Pass Marks : 32 **Duration: 3 Hours**

The figures in the margin indicate full marks for the questions

Answers the following questions:

1. Choose and rewrite the correct answer: $1 \times 3 = 3$

a) If S is any closed surface enclosing a volume V and $\vec{F} = ax\hat{i} + by\hat{j} + cz\hat{k}$, then the value of $\iint_S \vec{F}.\hat{n}\,dS$ is (Use Gauss Divergence theorem)

i) $abcV$ ii) $(a+b+c)V$

iii) $(ab+bc+ca)V$ iv) $\pi abcV$.

b) The semi-vertical angle of a right circular cone admitting sets of three mutually perpendicular generators is

i) $\tan^{-1}\sqrt{2}$ ii) $-\tan^{-1}\sqrt{2}$

iii) $-\tan^{-1}\sqrt{\frac{1}{2}}$ iv) $\tan^{-1}\sqrt{\frac{1}{2}}$.

c) Section of the conicoid $ax^2 + by^2 + cz^2 = 1$ on the plane $lx + my + nz = p$ will be hyperbola if

i) $bcl^2 + cam^2 + abn^2 = 1$

ii) $bcl^2 + cam^2 + abn^2 \neq 1$,

iii) $bcl^2 + cam^2 + abn^2 > 1$

iv) $bcl^2 + cam^2 + abn^2 < 1$

2. Write very short answers for each of the following: $1 \times 6 = 6$

a) Define a vector valued function.

b) If the vectors $\vec{a}$ and $\vec{b}$ are irrotational, then show that $\vec{a} \times \vec{b}$ is a solenoidal vector.

c) Write the equation of the cone with vertex at the origin and which pass through the curves $ax^2 + by^2 = 2z$, $lx + my + nz = p$.

d) What surface is represented by the equation $2x^2 - 6y^2 - 3z^2 = 1$?

e) Define Principal planes of a central conicoid.

f) How many normal can be drawn through any given point to a central conicoid?

three

3. Write short answers (any two) of the following: $3 \times 5 = 15$

a) State and prove the necessary and sufficient condition that a vector $\vec{a}$ have a constant direction.

b) Find the equation of the sphere through the origin and making intercepts a, b, c on the co-ordinate axes.

c) Find the equation of the right circular cone with its vertex at the origin, axis along Z-axis and semi-vertical angle.

d) Find the equation to the tangent planes to the conicoid $2x^2 - 6y^2 + 3z^2 = 5$ which pass through the line $x + 9y - 3z = 0 = 3x - 3y + 6z - 5$.

e) Show that the plane $2x - 4y - z + 3 = 0$ touches the paraboloid $x^2 - 2y^2 = 3z$. Also find the co-ordinate of the point of contact.

4. Answer any two of the following: $4 \times 5 = 20$

a) If $\vec{a}', \vec{b}', \vec{c}'$ are the reciprocal systems of vectors $\vec{a}, \vec{b}, \vec{c}$ respectively, prove that $[\vec{a}\vec{b}\vec{c}][\vec{a}'\vec{b}'\vec{c}'] = 1$.

b) Suppose $\nabla \times \vec{A} = 0$. Evaluate $\nabla \cdot (\vec{A} \times \vec{r})$.

c) Prove that $2x^2 + 2y^2 + 7z^2 - 10yz - 10zx + 2x + 2y + 26z - 17 = 0$ represents a cone with vertex at $(2,2,1)$.

d) A tangent plane to the conicoid $ax^2 + by^2 + cz^2 = 1$, meets the co-ordinate axes in P, Q and R. Find the locus of the centroid of the triangle PQR.

e) Prove that the enveloping cylinders of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, whose generators are parallel to the lines $x = 0, \pm\frac{y}{\sqrt{a^2 + x^2}} = \frac{z}{c}$ meet the plane $z = 0$ in circles.

5. Answer any two of the following: **$6 \times 2 = 12$**

a) Verify Stokes' theorem for the vector function $\vec{A} = 3y\hat{i} - xz\hat{j} + yz^2\hat{k}$ and the surface S of the paraboloid $x^2 + y^2 = 2z$ bounded by $z = 2$.

b) Verify Green's theorem in plane for $\oint_C \{(x^2 + xy)dx + xdy\}$, where C is the curve enclosing the region bounded by $y = x^2$ and $y = x$.

c) Suppose that the surface S has projection on the xy-plane. Show that $\iint_S \vec{A}\cdot\hat{n}\,dS = \iint_R \vec{A}\cdot\hat{n}\frac{dxdy}{|\hat{n}.\hat{k}|}$.

6. Answer any two of the following: **$6 \times 2 = 12$**

a) Find the centre and the radius of the circle $x + 2y + 2z = 15, x^2 + y^2 + z^2 - 2y - 4z = 11$.

b) Find the equation of the cone whose vertex is the point (α, β, γ) and whose generators intersect the conic

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0, z = 0.$$

c) Find the equation of the right circular cone whose vertex is $(3, 2, 1)$, axis is the line $\frac{x-3}{4} = \frac{y-2}{1} = \frac{z-1}{3}$ and semi-vertical angle 30°.

7. Answer any two of the following: **$6 \times 2 = 12$**

a) Find the equation of the tangent plane at the point (α, β, γ) of the central conicoid $ax^2 + by^2 + cz^2 = 1$.

b) Find the locus of the point of intersection of three mutually perpendicular tangent planes to the central conicoid

$$ax^2 + by^2 + cz^2 = 1.$$

c) Find the equation of the tangent plane to the paraboloid $\frac{x^2}{5} - \frac{y^2}{3} = 2z$ parallel to the plane $2x - 3y + z = 0$.

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