

DHANAMANJURI UNIVERSITY

Examination- 2023 (June)

Four year course B.Sc./B.A. 2nd Semester

Name of Programme : B.Sc./B.A. Mathematics
Paper Type : Core VI (Theory)
Paper Code : CMA-106
Paper Title : Vector analysis and solid Geometry
Full Marks : 40
Pass Marks : 16 **Duration: 2 Hours**

The figures in the margin indicate full marks for the questions

Answer any four of the following questions.

1. (a) If $\vec{A} = 3xyz^2\hat{i} + 2xy^3\hat{j} - x^2yz\hat{k}$, find the $\text{div}\vec{A}$ at the point $(1, -1, 1)$.
 (b) Show that $\vec{\nabla} \cdot (\vec{a} \times \vec{r}) = \vec{r} \cdot (\vec{\nabla} \times \vec{a})$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. 4+6=10
2. (a) Write the statement of Green's theorem and Verify Green's theorem.

$$\int_C (3x^2 - 8y^2)dx + (4x - 6xy)dy$$
, where C is the boundary of the region defined by $y = \sqrt{x}$,
 $y = x^2$. 2+8=10
3. State Gauss divergence's theorem. Verify Gauss Divergence's theorem given that
 $\vec{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ and S is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$. 2+8=10
4. (a) Find the equation of the sphere through the points $(0, 0, 0), (1, -1, 0), (2, 0, -2)$ and $(0, 1, 2)$.
 (b) Find the equation of the Sphere on which the circle given by $x + y + z = 0$ and
 $x^2 + y^2 + z^2 = 9$ is a great circle. 5+5=10
5. (a) Derive the equation of a cone with a given vertex (α, β, γ) and a conic
 $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0, z = 0$ as guiding curve.
 (b) Find the equation of the right circular cone whose vertex is (x_1, x_2, x_3) axis is
 $\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$ and semi-vertical angle is θ . 5+5=10
6. (a) Write the condition that a second degree equation
 $ax^2 + 2hxy + by^2 + cz^2 + 2fyz + 2gzx + 2ux + 2vy + 2wz + d = 0$ may represent a cone.
 (b) Find the equation of the cone with vertex $(5, 4, 3)$ and $3x^2 + 2y^2 = 6, y + z = 0$
 is the base. 2+8=10
7. (a) Find the equation of right circular cylinder whose axis is $\frac{x - \alpha}{l} = \frac{y - \beta}{m} = \frac{z - \gamma}{n}$ and radius r .
 (b) Find the equation of the cylinder generated by lines parallel to a fixed line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$,
 the guiding curve being the conic $z = 0, ax^2 + by^2 = 1$ 5+5=10
8. a) Name the central conicoid represented by $ax^2 + by^2 = 2z$ when the constants a and b are of same sign.
 b) Write down the equation of a nyperbolic paraboloid.
 c) Find the condition that the plane $lx = my = nz = p$ may be a tangent plane to the conicoid
 $ax^2 + by^2 + cz^2 = 1$. 2+2+6=10
