DHANAMANJURI UNIVERSITY

Examination- 2026 (June)

Name of Programme : B.A./ B.Sc. Mathematics
Semester : 2nd
Paper Type : Core
Paper Code : CMA-105
Paper Title : Differential Equations
Full Marks : 80
Pass Marks : 32 **Duration: 3 Hours**

The figures in the margin indicate full marks for the questions.

Answer all the questions:

1. Choose the correct answer from the following and rewrite it: **$1 \times 3 = 3$**

a) The degree of the differential equation $y = \sqrt{x}\frac{dy}{dx} + \frac{2}{\frac{dy}{dx}}$ is

i) 1 ii) 2
iii) 3 iv) 4

b) The solution of $\left(\frac{dy}{dx}\right)^3 = x^4$ is

i) $y = \frac{4}{3}x^{\frac{1}{3}} + c$ ii) $y = \frac{3}{7}x^{\frac{7}{3}} + c$
iii) $y = \frac{7}{3}x^{\frac{7}{3}} + c$ iv) $y = \frac{1}{3}x^{\frac{7}{3}} + c$

c) The particular integral of $(D^2 + a^2)y = \sin ax, D \equiv \frac{d}{dx}$ is

i) $\frac{x}{2a}\sin ax$ ii) $-\frac{x}{2a}\sin ax$
iii) $\frac{x}{2a}\cos ax$ iv) $-\frac{x}{2a}\cos ax$

2. Write very short answer for each of the following questions: **$1 \times 6 = 6$**

a) Define integrating factor of a differential equation.

b) When is a family of curves said to be self-orthogonal?

c) Write the solution of the differential equation
$y = x\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^3$.

d) Find the solution of $y^2 = \left(\frac{py}{x}\right)x^2 + \left(\frac{py}{x}\right)^2$, where $p = \frac{dy}{dx}$.

e) Evaluate Wronskian of the functions $y_1(x) = x$ and $y_2(x) = xe^x$.

f) Find the primitive of $(D^2 - 2D + 5)^2 y = 0$.

3. Write short answer for each of the following: $3 \times 5 = 15$

a) Find the value of the constant λ such that the equation $(2xe^y + 3y^2)\frac{dy}{dx} + (3x^2 + \lambda e^y) = 0$ is exact. Further, using the value of λ, solve the equation.

b) Solve: $(1 + y^2)dx = (\tan^{-1} y - x)dy$.

c) Solve: $\frac{dy}{dx} = e^{x-y}(e^x - e^y)$.

d) The population of a country was 23 million in 1990 and 27 million in 1995. Predict the population of the country in 2000.

e) Solve: $(x^3D^3 + 3x^2D^2 + xD)y + y = \log x + x$, where $D \equiv \frac{d}{dx}$.

4. Answer the following questions: $4 \times 5 = 20$

a) Solve the simultaneous equations:
$\frac{adx}{(b-c)yz} = \frac{bdy}{(c-a)zx} = \frac{cdz}{(a-b)xy}$

b) Solve: $y = 2px + y^2p^3$, where $p = \frac{dy}{dx}$.

c) Find an expression for the time taken to double the population for exponential growth without limitations.

d) Solve: $(D^2 + 1)y = \sec x$.

e) Solve: $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x^2$ using the method of undetermined coefficients for finding particular integral.

5. Answer any two of the following questions: $6 \times 2 = 12$

a) Solve: $(y^2 + 2x^2y)dx + (2x^3 - xy)dy = 0$.

b) Define total differential equation. Show that the total differential equation $Pdx + Qdy + Rdz = 0$ is integrable if $P\left(\frac{\delta Q}{\delta z} - \frac{\delta R}{\delta y}\right) + Q\left(\frac{\delta R}{\delta x} - \frac{\delta P}{\delta z}\right) + R\left(\frac{\delta P}{\delta y} - \frac{\delta Q}{\delta x}\right) = 0$

c) Find the orthogonal trajectories of the family of co-axial circles $x^2 + y^2 + 2gx + c = 0$, where g is the parameter

6. Answer any two of the following questions: $6 \times 2 = 12$

a) Solve the differential equation $y = x - 2ap + ap^2$, where $p = \frac{dy}{dx}$. Find the singular solution and interprete it geometrically.

b) The rate at which radioactive nuclei decay is proportional to the number of such nuclei that are present in a given sample. Half of the original number of radioactive nuclei have undergone disintegration in a period of 1500 years. What percentage of the original radioactive nuclei will remain after 4500 years?

c) A body at an unknown temperature is placed in a room which is held at a constant temperature of 30^oF. If after 10 minutes the temperature of the body is 0^oF and after 20 minutes the temperature of the body is 15^oF, then find the unknown initial temperature.

7. Answer any two of the following questions: $6 \times 2 = 12$

a) Solve: $(x^3D^3 + 2x^2D^2 + 2)y = 10\left(x + \frac{1}{x}\right)$, where $D \equiv \frac{d}{dx}$.

b) Show that two solutions $y_1(x)$ and $y_2(x)$ of the equation $a_0(x)\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_2(x)y = 0, a_0(x) \neq 0, x \in (a, b)$ are linearly dependent if and only if their Wronskian is identically zero.

c) Using the method of variation of parameter, solve $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x^2e^x$.

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