DHANAMANJURI UNIVERSITY

Examination- 2025 (June)

Four-year course B.A/B.Sc. 2nd Semester (NEP)

Name of Programme	:	**B.A. / B.Sc. Mathematics (Honours)**
Paper Type	:	**CORE (Theory)**
Paper Code	:	**CMA-105**
Paper Title	:	**Differential Equations**
Full Marks	:	**80**
Pass Marks	:	**32** **Duration: 3 Hours**

The figures in the margin indicate full marks for the questions.

1. Choose and rewrite the correct answer from each of the following: **1×3=3**

a) The order of the differential equation of the family of curves

$y = ACosmx + BSinmx$ where m is fixed and A,B are arbitrary constants is

A) 1

B) 2

C) 3

D) 4

b) Clairaut's form of the equation $SinpxCosy = CospxSiny + p$ is

A) $y = px + Sin^{-1} p$

B) $y = px - Cos^{-1} p$

C) $y = px - Sin^{-1} p$

D) $y = px + Cos^{-1} p$

c) $y = e^x$ is a solution of $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = 0$ if

A) $P + xQ = 0$

B) $1 - P + Q = 0$

C)) $1 + \frac{P}{a} + \frac{Q}{a^2} = 0$

D) $1 + P + Q = 0$

2. Write very short answer for each of the following. **1×6=6**

a) Find the integrating factor of $\cos x \frac{dy}{dx} + y = \sin x$

b) Solve : : $p^2 - 9p + 18 = 0$

c) Define mathematical model.

d) Find the particular integral of $(D^2 - 3D + 2)y = e^x$

e) Define Linear Independence of solution of an equation.

f) Is $x + \frac{1}{x}$ an integral of the equation $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$?

3. Write short answer for each of the following: **5×4=20**

a) If $\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)$ is a function of x alone(say) f(x), then prove that

$e^{\int f(x)dx}$ is an integrating factor of $Mdx + Ndy = 0$

b) Solve: $\frac{dx}{z-y} = \frac{dy}{x-z} = \frac{dz}{y-x}$

c) Solve $x = y + p^2$

d) Prove that $1, x, x^2$ are linearly independent.

4. Write short answer for each of the following: **3×5=15**

a) Solve: $\frac{dz}{dx} + \frac{z}{x}\log z = \frac{z}{x^2}(\log z)^2$,, given that z=1 when x=1

b) Show that the equation

$(yz + xyz)dx + (zx + xyz)dy + (xy + xyz)dz = 0$ is integrable.

c) The magnitude of the velocity of a particle moving along the x-axis is given by the equation $V = \frac{x}{4}$, where V is metres per second and x is metres. When t=0, the particle is 2metres to the right of the origin. Find the position of the particle when t=3 seconds.

d) Solve: $e^{p-y} = p^2 - 1$

e) Solve: $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^4$

5. Answer any two questions: **6×2=12**

a) Find the necessary and sufficient condition that the equation $Mdx + Ndy = 0$ may be exact.

b) Verify that the equation

$(y^2 + yz)dx + (xz + z^2)dy + (y^2 - xy)dz = 0$ is integrable and solve it.

c) Find the orthogonal trajectories of the cardiod

$\gamma = a(1 - Cos\,\theta)$,

where 'a' is a parameter.

6. Answer any two questions: **6×2=12**

a) Find the complete primitive and singular solution of the equation $y = px + \sqrt{1 + p^2}$, interpret the result.

b) If the population of a state doubles in 40 year, in how many year will it be triple under the assumption that the rate of increase is proportional to the number of inhabitants.

c) Radium disappears at a rate proportional to the amount present. If 5% of the original amount disappears in 50 years, how much will remain at the end of 100 year.

7. Answer any two questions: **6×2=12**

a) Using the method of variation of parameters, solve

$$y'' + 4y = \sin\ x$$

b) Solve: $x^2\frac{d^2y}{dx^2} - 2x(1+x)\frac{dy}{dx} + 2(1+x)y = x^3$

c) Solve by the methodof undetermined coefficients, the equation $(D^2+1)y = 10e^{2x}$ with conditions $y=0$ and $Dy=0$ when $x=0$

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