DHANAMANJURI UNIVERSITY

Examination- 2026 (June)

Name of Programme : **B.A./ B.Sc. Mathematics**
Semester : **2nd**
Paper Type : **Core**
Paper Code : **CMA-104**
Paper Title : **Real Analysis**
Full Marks : **80**
Pass Marks : **32**
Duration: 3 Hours

The figures in the margin indicate full marks for the questions.

Answer all the questions:

1. Choose the correct answer from the following and rewrite it: $1 \times 3 = 3$

a) Which of the following set is a closed set?

i) $[2,7) \cap (4,8]$ ii) $(2,4) \cap (7,8)$

iii) $[2,4) \cup (7,8]$ iv) $(2,7] \cup [4,8)$

b) For the sequences $\{u_n\}$ where

$u_n = \{$ 2 if n = even and greater than 10
-10 if n = even and less than 12
7 if n = odd and greater than 9
5 if n = odd and less than 11

Then $\underline{\lim} u_n$ and $\overline{\lim}\ u_n$ are respectively

i) -10 and 7 ii) 7 and -10

iii) 2 and 7 iv) 7 and 2

(The symbols have their usual meaning)

c) The geometric infinite series $1+x+x^2+\ldots+x^{n-1}+\ldots$, is convergent when

i) $x < -1$ ii) $x = 1$

iii) $x > 1$ iv) $|x| < 1$

2. Write very short answer for each of the following: $1 \times 6 = 6$

a) Give an example of a non – empty set which has no infimum.

b) Define limit point of a given set S of real numbers.

c) Define an open cover of a set.

d) Give an example of a sequence which has an infinite number of limit points.

e) Given an example of an oscillatory sequence.

f) Give an example of a conditionally convergent series.

3. Write short answer for each of the following: $3 \times 5 = 15$

a) Define a Countable set. Give an example of countable set which has infinite number of elements. Is the set of irrational numbers countable?

b) Show that every open set is a union of open intervals.

c) Show that the sequence $\{u_n\}$ where
$u_n = \frac{3}{1+n} + \frac{3}{2+n} + \ldots + \frac{3}{n+n}$
is convergent.

d) Prove that a necessary condition for the convergence of a series $\sum u_n$ is $\lim_{n\to\infty} u_n = 0$. By using an example, show that $\lim_{n\to\infty} u_n = 0$ is not a sufficient condition for the convergence of $\sum u_n$.

e) Show that the series $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots$
is convergent and the sum is 1.

4. Answer each of the following questions: $4 \times 5 = 20$

a) State Archimedean property of $\mathbb{R}$ and prove the same.

b) Define subsequence of a given sequence. If the sequence $\{u_n\}$ has a limit l, then show that any subsequence of $\{u_n\}$ has the same limit l.

c) Prove that a bounded monotonically increasing sequence converges to its supremum.

d) Sate Cauchy root test and prove the same.

e) Test the Convergence of the series $\sum \frac{n^2-1}{n+1} x^n$, where x > 0.

5. Answer any two of the following questions: **6 × 2 = 12**

a) State Bolzano – Weierstrass theorem for set and prove the same.

b) Define an open set. Show that the union of arbitrary family of open sets is open. Further give an example of infinite collection of open sets whose intersection is

i) not an open set and

ii) an open set.

c) State and prove Heine Borel Theorem.

6. Answer any two of the following questions: **6 × 2 = 12**

a) Define limit point of a sequence .Prove that every bounded sequence has a limit point.

b) State and prove nested intervals theorem.

c) Prove that the sequence $\{u_n\}$ defined by $u_1 = \sqrt{2}$ and $u_{n+1} = \sqrt{2u_n}$ for n > 1 , converges to 2.

7. Answer any two of the following questions: **6 × 2 = 12**

a) Show that the series $\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \ldots + \frac{1}{n^p} + \ldots$ is convergent if p > 1 and divergent if p ≤ 1

b) State D'Alembert's ratio test and prove the same.

c) State Leibnitz theorem for alternating series and prove the same.

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