DHANAMANJURI UNIVERSITY

Examination- 2025 (June)

Four-year course B.A/B.Sc. 2nd Semester (NEP)

Name of Programme	:	**B.A. / B.Sc. Mathematics (Honours)**
Paper Type	:	**CORE (Theory)**
Paper Code	:	**CMA-104**
Paper Title	:	**Real Analysis**
Full Marks	:	**80**
Pass Marks	:	**32**

Duration: 3 Hours

The figures in the margin indicate full marks for the questions.
Answer all the questions

1. Choose the correct answer from the following and rewrite: **1×3=3**

a) Let A = (2, 7) and B =[10, 12) U (13 , 17]. Then the derived set of AUB is

i) [2, 7] U [10 , 17]

ii) [2, 10] U [12 , 17]

iii) [2 , 7] U [10 , 12] U [13 , 17]

iv) (2, 7) U (10 , 12) U (13 , 17)

b) If the sequence $\{u_n\}$ is defined as

u_n = 2 if n = 4k - 1,
-7 if n = 4k − 2
3 if n = 4k − 3
-3 if n = 4k , k ≥ 1.

Then $\underline{\lim}\, u_n$ and $\overline{\lim}\, u_n$ are respectively

i) −3 and 3

ii) 2 and 3

iii) − 7 and − 3

iv) - 7 and 3

(The symbols have their usual meaning

c) The infinite series $\frac{1}{1^p}+\frac{1}{2^p}+\frac{1}{3^p}+\dots+\frac{1}{n^p}+\dots$ is convergent, if

i) $P>0$

ii) $P\leq 1$

iii) $P=0$

iv) $P>1$

2. Write very short answer for each of the following questions: 1×6=6

a) When is a set said to be countable?

b) State order completeness of real numbers.

c) Define a closed cover.

d) Define a Cauchy sequence.

e) Define a subsequence of a given sequence.

f) State D' Alembert ratio test.

3. Write short answer for each of the following questions : 3×5=15

a) Show that every open interval is an open set.

b) Show by means of a suitable example that arbitrary union of closed sets need not be closed. Also give an example of an arbitrary family of closed sets whose union is also closed.

c) Give an example of

i) an oscillatory sequence.

ii) a convergent sequence and

iii) a divergent sequence.

d) Test the convergence of the series

$$1+\frac{2}{5}x+\frac{6}{9}x^2+\frac{14}{17}x^3+\dots+\frac{2^n-2}{2^n+1}x^{n-1}+\dots$$

for $x>0$

e) Define absolute convergence and conditionally convergence series and give an example of a conditionally convergence series..

4. Answer each of the following questions: **4×5=20**

a) Define a limit point of a set. Using the definition of limit points, show that the union of two closed sets is a closed set.

b) Show that the sequence $\{u_n\}$ where

$$u_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \dots + \frac{1}{n!},$$

converges to e.

c) Show that the sequence $\{u_n\}$ defined by

$u_1 = \sqrt{2}$

And $u_{n+1} = \sqrt{2u_n}$ for $n \geq 1$,

converges to 2.

d) State Cauchy root test and applying the same to show that the series

$$\frac{1}{2} + \frac{2}{3}x + \left(\frac{3}{4}\right)^2 x^2 + \left(\frac{4}{5}\right)^3 x^3 + \dots \infty,$$

is convergent for all values of $x > 0$.

e) Test the convergence of the series

$$\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \frac{x^6}{5\sqrt{4}} + \cdots \infty$$

5. Answer any two questions: **6×2=12**

a) Define an open set. Show that every open set is a union of open intervals. Also show that the union of arbitrary family of open sets is open.

b) A set is closed if and only if its complement is open. Prove it.

c) Let S be a closed and bounded set of real numbers. Then prove that each open cover of S has a finite sub-cover

6. Answer any two questions: **6×2=12**

a) State and prove Cauchy convergence criterion for sequence.

b) Define a monotonic decreasing sequence and give an example of it . Also, show that a bounded and monotonically decreasing sequence converges to its infimum.

c) Show that every bounded sequence has a limit point.

7. Answer any two questions : **6×2=12**

a) State and prove Cauchy's general principle of convergence of series.

b) The series

$1 + a + a^2 + \dots + a^{n-1} + \dots,$

Converges, if $|a| < 1$ and its sum is $\frac{1}{1-a}$. It diverges if , $a \geq 1$, oscillates finitely if $a = -1$ and oscillates infinitely if $a < -1$. Prove it.

c) State and prove Leibnitz theorem for alternating series.

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