DHANAMANJURI UNIVERSITY
Examination- 2023 (June)
Four-year course B.Sc./B.A. 2nd Semester

Name of Programme	:	B.Sc./B.A. Mathematics
Paper Type	:	Core IV(Theory)
Paper Code	:	CMA-104
Paper Title	:	Real Analysis
Full Marks : 40		
Pass Marks : 16		Duration: 2 Hours

The figures in the margin indicate full marks for the questions
Answer any four of the followinf questions:

Answer the following: 10×4=40

1. a) Prove that every subset of a countable set is countable.

   b) State and prove Archimedean property of $\mathbb{R}$.

2. a) Define an open set. Show that every open interval is an open set.

   b) The union of arbitrary family of open sets is open. Prove it.

3. State and prove Bolzano-Weierstrass theorem for set.

4. Show that a set is closed if and only if its complement is open. Further using this, show that the intersection of arbitrary collection of closed sets is a closed set.

5. State Heine Borel theorem and prove the same.

6. Show that the infinite series $\dfrac{1}{1^p} + \dfrac{1}{2^p} + \cdots + \dfrac{1}{n^p} + \cdots$, is convergent, if $p > 1$, and divergent if $p \leq 1$.

7. State and prove D' Alembert's ratio test.

8. Define an alternating series. State and prove Leibnitz's theorem. Show that the series $1 - \dfrac{1}{2} + \dfrac{1}{3} - \dfrac{1}{4} + \cdots$ is convergent.

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