DHANAMANJURI UNIVERSITY
DECEMBER 2025

Name of Programme : B.A./B.Sc. Mathematics
Semester : 1st
Paper Type : Core
Paper Code : CMA-103
Paper Title : Analytical Geometry
Full Marks : 80
Pass Marks : 32
Duration: 3 Hours

The figures in the margin indicate full marks for the questions.
Answer all the questions.

1. Choose and rewrite the correct answer: $1 \times 3 = 3$

(a) Which of the following represents the transformed equation of the parabola $y^2 = 4ax$ when the origin is transferred to the point $(a, 0)$?

(i) $y^2 = 4a(x - a)$ (ii) $y^2 = 4(x - a)$

(iii) $y^2 = 4a(x + a)$ (iv) $y^2 = 4(x + a)$

(b) The equation of the diameter of the conic $4x^2 + 6x - 5y^2 = 1$ conjugate to the diameter $y = 2x$ is given by

(i) $10x - 7y = 0$ (ii) $10x + 7y = 0$

(iii) $x - 7y = 0$ (iv) $10x + y = 0$

(c) If $\frac{l}{r} = e\cos\theta + \cos(\theta - \alpha)\sec\beta$ is the equation of the chord to the conic $\frac{l}{r} = 1 + e\cos\theta$ then equation of the tangent is obtained by putting

(i) $\alpha = \beta$ (ii) $\alpha = 0$ (iii) $\beta = 0$ (iv) $r = 0$

2. Write very short answer for each of the following: $1 \times 6 = 6$

(a) Define invariants of the transformation.

(b) What is the condition for the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ to represent a pair of lines?

(c) Write down the condition that the line $y = mx + c$ is a tangent to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

(d) Find the equation of the polar of the origin with respect to the conic

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0.$$

(e) Write down the equation of the tangent at the point on the conic $\frac{l}{r} = 1 + e\cos\theta$ whose vectorial angle is α.

(f) Determine the nature of the conic $\frac{3}{r} = 2 + 2\cos\theta$.

3. Write short answer for each of the following: $3 \times 5 = 15$

(a) Establish the transformed formulae for the change of origin without change of direction of axes.

(b) If the pair of lines $x^2 - 2pxy - y^2 = 0$ and $x^2 - 2qxy - y^2 = 0$ be such that each pair bisects the angles between the other pair, prove that $pq + 1 = 0$.

(c) Prove that the line $lx + my = n$ is a normal to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

if $\frac{a^2}{l^2} + \frac{b^2}{m^2} = \frac{(a^2-b^2)^2}{n^2}$.

(d) If $y = x\sin\alpha + a\cos\alpha$ be a tangent to the circle $x^2 + y^2 = a^2$, then prove that $\cos^2\alpha = 1$

(e) Prove that the equation $(x-p)^2+2h(x-p)(y-q)-(y-q)^2=0$ represents a pair of perpendicular lines.

4. **Answer any five of the following:** $4\times5=20$

(a) Choose a new origin (h,k) without changing the directions of the axes, such that the equation $5x^2-2y^2-30x+8y=0$ may reduce to the form $Ax'^2+By'^2=1$.

(b) Show that $\frac{a}{h}=\frac{h}{b}=\frac{g}{f}$ is the condition for the equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ to represent a pair of parallel lines.

(c) Give the definition of the polar of a point P with respect to the circle. If the polar of a point P with respect to a circle passes through Q prove that the polar of Q passes through P.

(d) Define a parabola. Derive the standard form of parabola .

(e) When are the two lines said to be conjugate to each other with respect to a conic? Obtain the conditions the lines $lx+my+n=0$ and $l'x+m'y+n'=0$ may be conjugate lines with respect to the conic $ax^2+2hxy+by^2+2gx+2fy+c=0$.

(f) Write down the polar equation of a conic with focus as the pole and the line joining the focus to the corresponding vertex as the initial line. Prove that the semi-latus rectum of any conic is the harmonic mean between the segments of any focal length.

5. **Answer any two of the following:** $6\times2=12$

(a) Prove that a homogeneous second degree equation of the form $ax^2+2hxy+by^2=0$ represents a pair of straight lines through the origin.

(b) Find the transformed equation of the form $(x+2y+4)(2x-y+5)=25$ when the two perpendicular lines $x+2y+4=0$ and $2x-y+5=0$ are taken as co-ordinate axes.

(c) Show that the equation
$abx^2 + (a^2 + b^2)xy + aby^2 + ab(a - b)(x - y) - a^2b^2 = 0$
represents two straight lines equidistant from the origin.

6. Answer any two of the following: $6 \times 2 = 12$

(a) Find the equation of the chord of the circle $x^2 + y^2 = a^2$ in terms of its middle point (x', y').

(b) Reduce the equation $3x^2 - 6xy - 5y^2 - 6x + 22y - 17 = 0$ to the standard form of conic.

(c) A chord of the parabola subtends a right angle at vertex. Prove that the tangents at the extremities of the chord intersect on the line $x + 4a = 0$.

7. Answer any two of the following: $6 \times 2 = 12$

(a) Establish the equation of a chord of the conic $\frac{l}{r} = 1 + e\cos\theta$ joining the two points on the conic, whose vectorial angles $(\alpha + \beta)$ and $(\alpha - \beta)$.

(b) Find the equation of the chord of contact of tangent from a given point (r_1, θ_1) to the conic $\frac{l}{r} = 1 + e\cos\theta$.

(c) Prove that the shortest focal chord of a conic is the latus rectum.

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