DHANAMANJURI UNIVERSITY

Examination- 2024 (Dec)

Four-year course B.Sc./B.A. 1^{st} Semester

Name of Programme : B.Sc./B.A. Mathematics
Paper Type : Core III (Theory)
Paper Code : CMA-103
Paper Title : Analytic Geometry
Full Marks : 80
Pass Marks : 32 **Duration: 3 Hours**

The figures in the margin indicate full marks for the questions.

1. Choose and rewrite the correct answer: 1 × 3 = 3

a) The equation of bisectors of the angles between pair of lines is represented by:
i) $x^2 - y^2 = \frac{xy}{h}$, ii) $\frac{x^2-y^2}{a-b} = \frac{h}{xy}$,
iii) $\frac{x^2-y^2}{a-b} = \frac{xy}{h}$, iv) $y^2 - x^2 = \frac{h}{xy}$,

b) In a conic, where the equation is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$; the eccentricity is:
i) $e = 1$ ii) $e < 1$,
iii) $e > 1$, iv) $e \leq 1$,

c) The equation of the directix of the conic $\frac{l}{r} = 1 + ecos\theta$
i) $\frac{l}{r} = e\cos\theta$ ii) $r = \frac{l}{e}\cos\theta$
iii) $\frac{l}{r} = 1 - e\cos\theta$, iv) $r = \frac{l}{e}\sin\theta$,

2. Write very short answers for each of the following: 1 × 4 = 4

a) Write the condition that two lines are perpendicular to each other.
b) Write the condition for a pair of parallel straight lines.
c) Write the tangent equation to the circle $x^2 + y^2 = a^2$.
d) What are confocal conics?

3. Write short answers for each of the following: 3 × 7 = 21

a) Find the equation of the circle which passes through the points (5, 3), (6, -4) and (-1, -4). Also find the coordinates of the center of the circle.

b) Prove that the equation $6x^2 - 5xy - 6y^2 + 14x + 5y + 4 = 0$ represents a pair of perpendicular lines.

c) Find the point of intersection of a pair of lines represented by the equation $2x^2 - 5xy + 3y^2 - 2x + 3y = 0$.

d) Find the value of *k*, so that the equation $kx^2 + 3xy - 5y^2 + 7x + 14y + 3 = 0$ may represent a pair of straight lines .

e) Find the equation of the parabola whose focus is the point (-1, 1) and whose directrix is the straight line $x + y + 1 = 0$. Find also the length of the latus rectum.

f) Find the condition that the line $lx+my = n$ is a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$.

g) Find the condition that the line $\frac{l}{r} = A\cos\theta + B\cos\theta$ may be tangent to the conic $\frac{l}{r} = 1 + e\cos\theta$.

4. Answer any *four* of the following: 4 × 4 = 16

a) Find the angle between the lines represented by the general equation: $ax^2 + 2hxy + by^2 + 2gx + fy + c = 0$.

b) What are orthogonal circles? Derive the condition for orthogonality of the two circles.

c) Prove that the line $lx + my = n$ is a normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

d) Obtain the equation of the parabola whose focus is at the point (3, -2) and directrix is the line $2x - y = 0$.

e) Find the polar equation of a conic with its focus as the pole and its axis inclined at an angle α to the initial line.

5. Answer any *two* of the following: 6 × 2 = 12

a) Prove that the condition for pair of lines represented by a general equation of the second degree is $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$.

b) The equation of the pair of opposite sides of a rectangle are $x^2 - 7x + 6 = 0$ and $y^2 - 14y + 40 = 0$. Find the equations of its diagonals.

c) Show that the equation
$ab(x^2+y^2)+(a^2+b^2)xy+ab(a-b)(x-y)=a^2b^2$ represents two straight lines equidistant from the origin.

6. Answer any *two* of the following: 6 × 2 = 12

a) Find the locus of the poles of tangents to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with respect to the ellipse $\frac{x^2}{\alpha^2}+\frac{y^2}{\beta^2}=1$.

b) If e and e' be the eccentricities of a hyperbola and its conjugate, show that $\frac{1}{e^2}+\frac{1}{e'^2}=1$.

c) Prove that the line $2x+4y=9$ is a normal to the parabola $y^2=8x$. Find the coordinates of the foot of this normal.

7. Answer any *two* of the following: 6 × 2 = 12

a) Prove that the sum of the reciprocals of two perpendicular focal chords of a conic is constant.

b) Find the equation of the normal at the point of angle α on the conic $\frac{l}{r}=1+ecos\theta$.

c) Referring to the equation $\frac{x^2}{a^2+\lambda}+\frac{x^2}{b^2+\lambda}=1$ of confocals to an ellipse, show that two confocals cut at right angle.

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