DHANAMANJURI UNIVERSITY

DECEMBER 2025

Name of Programme : B.A./B.Sc. Mathematics
Semester : 1st
Paper Type : Core
Paper Code : CMA-102
Paper Title : Algebra
Full Marks : 80
Pass Marks : 32

Duration: 3 Hours

The figures in the margin indicate full marks for the questions.

1. Choose and rewrite the correct answer for each of the following: 1x3=3

(a) The exponential value of $sinh\, x$ is

(i) $\frac{e^{ix}-e^{-ix}}{2i}$ (ii) $\frac{e^{ix}+e^{-ix}}{2i}$

(iii) $\frac{e^{x}-e^{-x}}{2}$ (iv) $\frac{e^{x}+e^{-x}}{2}$

(b) If a, b, c are in harmonic progression, then

(i) $b=\frac{a+c}{2}$ (ii) $b=\frac{2ac}{a+c}$

(iii) $b=\frac{a+c}{2ac}$ (iv) $b=\frac{ac}{2(a+c)}$

(c) For every characteristic root of a Matrix,

(i) G.M. > A.M. (ii) G.M. $\geq$ A.M.

(iii) G.M. $\leq$ A.M. (iv) G.M. = A.M.

where G.M.,A.M.is Geometric Multiplicity and Algebraic Multiplicity respectively.

2. Write very short answer for each of the following questions: 1x6=6

(a) State De Moivre's theorem.

(b) Show that $\cosh x = \cos ix$.

(c) State fundamental theorem of algebra.

(d) Write a matrix which is symmetric but not Hermitian.

(e) What will be the characteristic roots of a Hermitian matrix?

(f) What will be rank of a matrix when its row rank is 2?

3. Write short answer for each of the following questions: 3x5=15

(a) Find the principal value of $\cosh^{-1} z$.

(b) Show that the equation $x^4 - 2x^3 - 1 = 0$ has two imaginary roots.

(c) Find the equation whose roots are equal to the roots of $x^3 - 9x^2 + 28x - 27 = 0$ each diminished by 3.

(d) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 4 & 6 & 2 \\ 1 & 2 & 3 & 2 \end{bmatrix}$ by reducing to Echelon form.

(e) Show that the vectors $\begin{bmatrix} 2 & 3 & -1 & -1 \end{bmatrix}$, $\begin{bmatrix} 1 & -1 & -2 & -4 \end{bmatrix}$, $\begin{bmatrix} 3 & 1 & 3 & -2 \end{bmatrix}$, $\begin{bmatrix} 6 & 3 & 0 & -7 \end{bmatrix}$ are linearly dependent.

4. Write answer for each of the following questions: 4x5=20

(a) If $a = \cos\theta + i\sin\theta$, $b = \cos\varphi + i\sin\varphi$, find the values $\cos(\theta+\varphi)$ and $\cos(\theta-\varphi)$ in terms of a, b.

(b) Evaluate $Log(\alpha + i\beta)$, where α and β are real.

(c) If $x > 0$, $y > 0$, $z > 0$ and $x + y + z = 1$, prove that

$$\frac{x}{2-x} + \frac{y}{2-y} + \frac{z}{2-z} \geq \frac{3}{5}.$$

(d) State and prove Holder's inequality.

(e) Show that every square A can be expressed uniquely as $P + iQ$ where P and Q are Hermitian matrices.

5. Answer any two of the following questions: 6x2=12

(a) Find all the values of $\sqrt[7]{1+i}$.

(b) State and prove Gregory's series.

(c) Find the sum of the infinite series

$$\sin\alpha + \frac{1}{2}\sin 2\alpha + \frac{1}{2^2}\sin 3\alpha + \cdots$$

6. Answer any two of the following questions: 6x2=12

(a) Solve the reciprocal equation $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$.

(b) Solve the biquadratic equation $x^4 - 10x^2 + 4x + 8 = 0$ by using Ferrari's method.

(c) Solve the equation $9x^3 - 6x^2 + 1 = 0$ by using Cardan's method.

7. Answer any two of the following questions: 6x2=12

(a) (Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 4 & 5 & 4 \end{bmatrix}$ by using the matrix equation $AX = B$.

(b) Find the Latent values of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. Find also the Latent vector/vectors corresponding to the smallest Latent value.

(c) Find the characteristic equation of the matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}.$$

Verify Cayley-Hamilton theorem for this matrix and hence obtained A^{-1}.

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