DHANAMANJURI UNIVERSITY

Examination- 2024 (Dec)

Four year course B.Sc./B.A. 1^{st} Semester

Name of Programme	:	**B.Sc./B.A. Mathematics**
Paper Type	:	**Core II(Theory)**
Paper Code	:	**CMA-102**
Paper Title	:	**Algebra**

Full Marks : 80

Pass Marks : 32 **Duration: 3 Hours**

The figures in the margin indicate full marks for the questions:
Answer all the question.

1. Choose and rewrite the correct answer for each of the following: **1 × 3 = 3**

a) The principal value of the amplitude or argument of a complex number $z = (\cos\theta + i\sin\theta)$ is the value of θ which satisfies the inequality

i) $-\pi \leq \theta \leq \pi$ ii) $-\pi < \theta \leq \pi$

iii) $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ iv) $-\frac{\pi}{2} < \theta \leq \frac{\pi}{2}$

b) Every diagonal element of a skew-Hermitian matrix is

i) Pure real number

ii) Pure imaginary number

iii) Either a pure imaginary number or zero

iv) Either a pure real number or zero

c) The equation whose roots are three times the roots of $x^3 + 2x^2 - 4x + 1 = 0$ is

i) $x^3 + 6x^2 - 36x + 27 = 0$

ii) $x^3 + 6x^2 + 36x + 27 = 0$

iii) $x^3 + 2x^2 + 4x + 1 = 0$

iv) $3x^3 + 6x^2 – 12x + 3 = 0$

2. Write very short answer for each of the following questions : **$1 \times 6 = 6$**

a) Find the remainder when $3x^4 + 11x^3 + 23x^2 + 21x - 10$ is divided by $x + 3$.

b) What do you mean by rank of a matrix ?

c) Express the complex number $0 + i$ in De Moivre's form.

d) Let $\alpha, \alpha + \beta, \alpha + 2\beta, ..., \{\alpha + (n - 1)\beta\}$ be n angles in A.P. Write the Expression for the sum of sines of these n angles in A.P.

e) State the Cauchy-Schwarz Inequality.

f) States Descartes rule of signs.

3. Write short answer for each of the following questions: **$3 \times 5 = 15$**

a) Find all the value of $(1 + i)^{\frac{1}{3}}$.

b) Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 2 & 4 & -4 & 7 \\ -1 & -2 & -1 & -2 \end{bmatrix}$ by reducing A to Echelon form.

c) If a, b, c are unequal and positive then prove that $\frac{bc}{b+c} + \frac{ca}{c+a} + \frac{ab}{a+b} < \frac{1}{2}(a + b + c)$.

d) Examine the linear independence or dependence of the following set of vectors $\{[1, 2- 1, 6], [3, 8, 9, 10], [2, -1, 2, -2]\}$.

e) Show that $x^3 - 7x + 2 = 0$ has one negative root, a positive root between 0 and 1 and another positive root greater than 1.

4. Write answer for each of the following questions: **$4 \times 5 = 20$**

a) State and prove Minkowski's inequality.

b) Find the value of the series $1 - \frac{2}{3!} + \frac{3}{5!} - \frac{4}{7!} + ...$

c) Express $\text{Log}\{\text{Log}(\cos\theta + i\sin\theta)\}$ in the form of $A + iB$

d) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

e) Find the equation whose roots are twice the reciprocals of the roots of $x^4 + 3x^3 - 6x^2 + 2x - 4 = 0$.

5. Answer any two of the following questions: $6 \times 2 = 12$

a) State and prove De Moivre's Theorem.

b) Prove that if $(a_1 + ib_1)(a_2 + ib_2)...(a_n + ib_n) = A + iB$ then

i) $\tan^{-1}\frac{b_1}{a_1} + \tan^{-1}\frac{b_2}{a_2} + ... + \tan^{-1}\frac{b_n}{a_n} = \tan^{-1}\frac{B}{A}$

ii) $(a_1^2 + b_1^2)(a_2^2 + b_2^2)...(a_n^2 + b_n^2) = A^2 + B^2$.

iii) If ϕ lies between $\frac{\pi}{4}$ and$\frac{3\pi}{4}$ show that $\phi = \frac{\pi}{2} - \cot\phi + \frac{1}{3}\cot^3\phi - \frac{1}{5}\cot^5\phi + ...$

6. Answer any two of the following questions: $6 \times 2 = 12$

a) Prove that if a and b are positive and unequal then $\frac{a^m+b^m}{2} > (\frac{a+b}{2})^m$,except when m lies between 0 and 1.

b) Solve $x^3 - 15x - 126 = 0$ by using Cardan's method.

c) If α, β, γ be the roots of the equation $x^3 + px^2 + qx + r = 0$ find the value of $\sum \frac{\beta^2 + \gamma^2}{\beta\gamma}$.

7. Answer any two of the following questions: $6 \times 2 = 12$

a) State and prove Cayley-Haminton theorem.

b) Define eigenvectors and eigenvalues of a matrix. Determine the eigenvalue and Eigen vectors of the matrix $A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$

c) Verify that the characteristic equation of the matrix $A = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 1 & 4 \\ 1 & -2 & 2 \end{bmatrix}$ is satisfied by A and hence obtain A^{-1}.

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