DHANAMANJURI UNIVERSITY

DECEMBER-2022

Name of Programme	**:**	**B.A. / B.Sc. Mathematics (Honours)**
Semester	**:**	**I**
Paper Type	**:**	**Core**
Paper Code	**:**	**CMA-102**
Paper Title	**:**	**Algebra**

Full Marks : 80

Pass Marks : 32 **Duration: 3 Hours**

The figures in the margin indicate full marks for the questions:
Answer all the questions.

1. Choose and rewrite the correct answer for each of the following questions: $1\times4=4$

a) The period of the hyperbolic sine is
   i) π ii) πi
   iii) 2π iv) $2\pi i$

b) The arithmetic mean of the numbers 1,3,5,...,$2n-1$ is
   i) n ii) $2n$
   iii) $3n$ iv) $4n$

c) If an algrbraic equation involves only odd powers of x and if all the co-efficient have positive signs, it has
   i) odd number of real positive roots
   ii) odd number of real negative roots
   iii) no real roots
   iv) the root zero and no other real roots

d) The matrix $\begin{bmatrix} i & -2 & 7 \\ -2 & 3i & 0 \\ 7 & 0 & 2i \end{bmatrix}$ is a

i) diagonal matrix ii)unit matrix
iii) symmetrix matrix iv) Hermitian matrix

2. Write very short answer for each of the following questions: 1× 10=10

a) Express $\sin x$ in ascending powers of x.

b) With validity, write the Gregory's series.

c) Prove that $\cos^2 x - \sinh^2 x = 1$.

d) If $a > b$, show that $a^a b^b > a^b b^a$.

e) State fundamental theorem of algebra.

f) State Descarte's rule of signs.

g) Define a reciprocal equation.

h) Define a skew Hermitian matrix.

i) Define a non singular matrix.

j) Show that the vectors; $u = (2,0,0), v = (0,1,0)$ and $w = (0,0,5)$ are linearly independent.

3. Write short answer for each of the following questions: $3 \times 10 = 30$

a) If $x + \dfrac{1}{x} = 2\cos\theta$, then prove that $x^n + \dfrac{1}{x^n} = 2\cos n\theta$.

b) If $x + iy = \sin(\alpha + i\beta)$, show that

i) $x^2 cosec^2\alpha - y^2 sec^2\alpha = 1$

ii) $x^2 sech^2\beta + y^2 cosech^2\beta = 1$

c) If $A + iB = \log(x + iy)$, show that $B = \tan^{-1}\left(\dfrac{y}{x}\right)$ and $A = \frac{1}{2}\log(x^2 + y^2)$.

d) If $a^2 + b^2 + c^2 = 1, x^2 + y^2 + z^2 = 1$ and a, b, c, x, y, z are unequal, show that $ax + by + cz < 1$.

e) If $a_1, a_2, ..., a_n$ and $b_1, b_2, ..., b_n$ are any real numbers, then show that $(a_1b_1 + a_2b_2 + ... + a_nb_n)^2 \leq (a_1^2 + a_2^2 + ... + a_n^2)(b_1^2 + b_2^2 + ... + b_n^2)$.

f) Show that the equation $x^4 - 2x^3 - 1 = 0$ has at least two imaginary roots.

g) Solve the cubic equation $x^3 - 7x^2 + 36 = 0$. Given that one root is double the another.

h) Every square matrix A can be expressed in one and only one way as $P+iQ$ where P and Q are Hermitian matrices.

i) Obtain the adjoint and reciprocal matrix of the matrix $A = \begin{bmatrix} 2 & 3 \\ -5 & -7 \end{bmatrix}$

j) Find the rank of the matrix $\begin{bmatrix} 0 & 1 & -3 & 1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$

4. Answer any two of the following questions: $6 \times 2 = 12$

a) Using De Moiverse's Theorem solve the equation $x^7 - x^4 + x^3 - 1 = 0$.

b) Express log sin$(\theta + i\phi)$ in the form $A + iB$.

c) Sum to infinity the series $\sin\theta + \frac{1}{2}\sin 2\theta + \frac{1}{2^2}\sin 3\theta + ...$

d) Find the sum to n terms of the infinite series $\cos^3\alpha + \cos^3(\alpha + \beta) + \cos^3(\alpha + 2\beta) + ...$

5. Answer any two of the following questions: $6 \times 2 = 12$

a) Show that the equation $x^3 - 4x + 2 = 0$ has exactly 3 real roots; 2 are positive, one lying between 0 and 1, other between 1 and 2 and a negative root in between -2 and -3.

b) If α, β, γ be the roots of the cubic equation $x^3 + px^2 + qx + r = 0$, find the value of $\dfrac{\beta^2+\gamma^2}{\beta+\gamma} + \dfrac{\gamma^2+\alpha^2}{\gamma+\alpha} + \dfrac{\alpha^2+\beta^2}{\alpha+\beta}$.

c) Solve by Cardan's method the cubic equation $x^3 - 18x - 35 = 0$.

d) Solve by Ferrari's method the biquadratic equation $x^4 + 12x - 5 = 0$.

6. Answer any two of the following questions: $6 \times 2 = 12$

a) Find the eigen values of the matrix

$$A = \begin{bmatrix} -2 & 2 & -3 \\ -2 & 1 & -6 \\ 2 & -2 & 0 \end{bmatrix}$$

and find the eigen vector corresponding to the largest eigen value.

b) Find the characteristic roots of the matrix

$$B = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

also find the characteristics vector corresponding to the smallest characteristics root.

c) State and prove Cayley-Hamilton theorem.

d) If A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix}$$

and I is the unit matrix of order 3, then show that $A^3 = pI + qA + rA^2$.

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