DHANAMANJURI UNIVERSITY
DECEMBER 2025

Name of Programme : B.A./B.Sc. Mathematics
Semester : 1st
Paper Type : Core
Paper Code : CMA-101
Paper Title : Calculus
Full Marks : 80
Pass Marks : 32 **Duration: 3 Hours**

The figures in the margin indicate full marks for the questions.

1. Choose and rewrite the correct answer for each of the following questions: 1X3=3

a) The limit of the function $e^x + e^{-x}$ when $x \to 0$ is
   i. 1
   ii. 0
   iii. 2
   iv. -1

b) The formula for the radius of curvature of a plane curve $y = f(x)$ is
   i. $\frac{\left\{1+\frac{dy}{dx}\right\}^{3/2}}{\frac{d^2y}{dx^2}}$
   ii. $\frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{1/2}}{\frac{d^2y}{dx^2}}$
   iii. $\frac{\left\{1+\frac{dy}{dx}\right\}^{1/2}}{\frac{d^2y}{dx^2}}$
   iv. $\frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2}}{\frac{d^2y}{dx^2}}$

c) The length of the arc of the curve along $y-axis$ between $y=c$ and $y=d$ is

i. $s=\int_c^d \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$

ii. $s=\int_c^d \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$

iii. $s=\int_c^d \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dy$

iv. $s=\int_c^d \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dx$

2. Write very short answer for each of the following questions: 1X6=6

a) State Rolle's theorem.
b) What is the curvature of a curve at a point?
c) Define a point of inflexion of a curve.
d) Find the volume of whole sphere generated by revolving a circle $x^2+y^2=a^2$ about $x-axis$.
e) Write the formula to calculate the surface area of solid form by rotating the curve $y=f(x)$ about $x-axis$.
f) Write a reduction formula for $\int_0^{\frac{\pi}{2}} \sin^m x\ \cos^n x\ dx$.

3. Write short answer for each of the following: 3X5=15

a) Find n^{th} derivative of $y=\cos 2x\ \cos x$.
b) Find all the relative extreme of the function $f(x)=x^3-3x$ by using the first derivative test.
c) Verify Euler's theorem for the function $u=x^3+y^3+3x^2+3xy^2$.
d) Find the relative maximum and relative minimum of the curve $f(x)=x^3-9x^2+5x+7$ using 2^{nd} derivative test.
e) Evaluate $\int_0^1 \int_x^{\sqrt{x}} (x^2+y^2)\,dy\,dx$.

4. Write short answer for each of the following questions: 4X5=20

a) Expand the function e^x in a finite series in power of x, with the remainder in cauchy's form.

b) Show that the function $f(x,y) = x^2 + 2xy + y^2 + x^3 + y^3 + x^7$ has neither a maximum nor minimum at the origin.

c) Find the radius of curvature at any point (r,θ) for the curve $r^2 \cos 2\theta = a^2$.

d) Examine the curve $y = x^3 - 9x^2 + 7x$, regarding its convexity or concavity. Determine its points of inflexion.

e) Find the area of the quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ between the major and minor axes.

5. Answer any two questions: 6X2=12

a) State and prove Lagrange's Mean Value theorem.

b) If $y = \sin^{-1} x$, then show that

i. $(1-x^2)y_2 - xy_1 = 0$

ii. $(1-x^2)y_{n+2} - 2(n+1)xy_{n+1} - n^2 y_n = 0.$

c) Evaluate $\lim_{x\to 0}(\cos x)^{\cos^2 x}$.

6. Answer any two questions: 6×2=12

a) Find the asymptotes of the curve $y^3 - 6xy^2 + 11x^2y - 6x^3 + y^2 - x^2 + 2x - 3y - 1 = 0.$

b) State and prove converse of Euler's theorem on Homogeneous functions of three variables.

c) If $u = \sin^{-1}\frac{x}{y} + \tan^{-1}\frac{x}{y}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0.$

7. Answer any two questions: 6X2=12

a) Find the length of the curve of the perimeter of the astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$.

b) Obtain reduction formula for $\int \sin^n x \, dx$ and hence find the reduction formula for $\int_0^{\frac{\pi}{2}} \sin^n x \, dx$, where n is a positive integer. Hence use this formula to evaluate $\int_0^{\frac{\pi}{2}} \sin^7 x \, dx$.

c) Find the volume and surface area of the solid obtained by revolving the circle $x^2+y^2 = a^2$ about x-axis.

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