DHANAMANJURI UNIVERSITY

Examination- 2024 (Dec)

Four year course B.Sc./B.A. 1^{st} Semester

Name of Programme	**:**	**B.Sc./B.A. Mathematics**
Paper Type	**:**	**Core I(Theory)**
Paper Code	**:**	**CMA-101**
Paper Title	**:**	**Calculus**

Full Marks : 80

Pass Marks : 32 **Duration: 3 Hours**

The figures in the margin indicate full marks for the questions.

1. Choose and rewrite the correct answer for each of the following: **$1 \times 3 = 3$**

a) If $y = e^{ax}$ then y_n is

i) e^{ax} ii)$a^n e^{ax}$

iii) $\frac{e^{ax}}{a}$ iv)ae^{ax}

b) If $y\dfrac{d^2y}{dx^2} > 0$ at $P(x, y)$, then the curve $y = f(x)$ at P with respect to x-axis is

i) Convex

ii) Concave

iii) Neither convex nor concave

iv) Either convex or concave

c) The maximum number of asymptotes of a curve of degree n is

i) $n - 1$ ii) $n + 1$

iii) n iv) n^2

2. Write very short answer for each of the following questions: **$1 \times 6 = 6$**

a) State Leibnitz's theorem.

b) Define limit of a function $f(x, y)$.

c) What is the condition for a point P to be a point of inflection on the curve $y = f(x)$?

d) Evaluate $\int_0^{\frac{\pi}{2}} \sin^6 x \, dx$.

e) Change the order of integration in the integral $\int_0^1 \int_0^x f(x,y)dydx$.

f) Find the length of the curve $y = 2x + 1$ on the interval [1,5].

3. Write short answer for each of the following: **3 × 5 = 15**

a) Find n^{th} derivative of the function $y = x^3 \sin x$.

b) Prove that the curve $y = \log x$ is convex to the foot of the ordinate in the range $0 < x < 1$ and concave where $x > 1$.

c) Write the steps of procedure for tracing of Cartesian curves.

d) Show that the function $f(x,y) = 3x^3 + 4x^2y - 3xy^2 - 4y$ is neither a maximum nor a minimum at (0,0).

e) Show that the area between $y = x^2$ and $x = y^2$ is $\frac{1}{3}$ sq. units.

4. Write short answer for each of the following questions: **4 × 5 = 20**

a) Evaluate: $\lim_{x \to 0} \dfrac{x - \sin x \cos x}{x^3}$.

b) Find all relative extrema of the function $f(x) = x^3 - 3x$ by using the first derivative test.

c) Find the radius of curvature at any point (x,y) for the curve $y = \frac{1}{2}a\left[e^{\frac{x}{a}} + e^{-\frac{x}{a}}\right]$.

d) Verify Euler's theorem for the function $u = \sin \frac{x^2+y^2}{xy}$.

e) Find the length of the perimeter of the circle $x^2 + y^2 = a^2$.

5. Answer any two of the following questions: **6 × 2 = 12**

a) State and prove Rolle's theorem.

b) Expand the function $\sin x$ in a finite series in powers of x, with the remainder in Lagrange's form.

c) Find the n^{th} derivative of the function $y = e^{3x} \sin 4x$.

6. Answer any two of the following questions: **6 × 2 = 12**

a) If $u = \log(x^2 + y^3 + z^3 - 3xyz)$, then show that $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = \dfrac{3}{x+y+z}$.

b) Examine the extreme values of $x^3 + y^3 - 3axy$.

c) Find the asymptotes of the curve
$y^3 - 6xy^2 + 11x^2y - 6x^3 + y^2 - x^2 + 2x - 3y - 1 = 0$.

7. Answer any two of the following questions: **6 × 2 = 12**

a) If $I_{m,n} = \int \sin^m x \cos^n x \, dx$ then show that $I_{m,n} = \frac{sin^{m+1}\ x\ cos^{n-1} x}{m+n} + \frac{n-1}{m+n} I_{m,n-2}$
Hence find a reduction formula for $\int_0^{\frac{\pi}{2}} \sin^m x \cos^n x \, dx$.

b) Show that $\displaystyle\int_0^1 dx \int_0^1 \frac{x-y}{(x+y)^3} dy \neq \int_0^1 dy \int_0^1 \frac{x-y}{(x-y)^3} dx$.

c) Find the volume and the surface area of the solid generated by revolving the arc of the astroid $x = a\cos^3\theta, y = a\sin^3\theta$ from $\theta = 0$ to $\theta = \frac{\pi}{2}$.

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