DHANAMANJURI UNIVERSITY

Examination- 2022 (Dec)

Four year course B.Sc./B.A. 1^{st} Semester

Name of Programme	**:**	**B.Sc./B.A. Mathematics(Honours)**
Paper Type	**:**	**Core**
Paper Code	**:**	**CMA-101**
Paper Title	**:**	**Calculus**

Full Marks : 80

Pass Marks : 32 **Duration: 3 Hours**

The figures in the margin indicate full marks for the questions:

Answer all the question.

1. Choose and rewrite the correct answer for each of the following: **$1 \times 5 = 5$**

a) The n^{th} derivative of $\frac{1}{x+a}$ is

i) $\frac{(-1)^n n!}{(x+a)^n}$ ii) $\frac{(-1)^{n+1}(n+1)!}{(x+a)^{n+1}}$

iii) $\frac{(-1)^n n!}{(x+a)^{n+1}}$ iv) $\frac{(-1)^{n+1} n!}{(x+a)^n}$

b) If $x = r\cos\theta, y = r\sin\theta$, then the value of $\frac{\delta\theta}{\delta x}$ is

i) $\frac{\sin\theta}{r}$ ii) $\frac{\cos\theta}{r}$

iii) $\cos\theta$ iv) $\frac{-\sin\theta}{r}$

c) The radius of curvature of the curve $y = log\sin x$ at the point (x, y) is

i) $cosec\ x$ ii) $sec\ x$

iii) $cot\ x$ iv) $tan\ x$

d) If $f(x, y) = \frac{x^3+y^3}{x-y}$,then the function $f(x, y)$ is a homogeneous of degree

i) 3 ii) 2

iii) 1 iv) 0

e) The area of the asteroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ is

i) $\frac{3}{4}\pi a^2$ ii) $\frac{4}{3}\pi a^2$

iii) $\frac{3}{8}\pi a^2$ iv) $\frac{8}{3}\pi a^2$

2. Write very short answer for each of the following questions: 1 × 6 = 6

a) State Leibnitz Theorem .

b) In the mean value theorem $f(b) = f(a) + (b-a)f'(c)$ if $f(x) = 2x^2, a = 0$ and $b = 2$, then find the value of c.

c) Define continuity of the function $f(x, y)$ at a point(a, b).

d) Write down the expression for the radius of the curvature of the Cartesian equation $y = f(x)$.

e) Write the value of $\int_0^{\frac{\pi}{2}} \cos^3 x dx$.

f) What is meant by concavity ?

3. Write short answer for each of the following questions: 3×9=27

a) Find the n^{th} derivative of $e^{ax+b} \sin x$.

b) Evaluate:$\lim_{x \to 0}(\frac{1}{(x^2)} - \frac{1}{(sin^2 x)})$.

c) Show that $\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2 + y^2}$ does not exist .

d) If $u = \tan^{-1}(\frac{y}{x})$ then show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y} = 0$.

e) Find the asymtotes of $x^3 + 2x^2y + xy^2 - x + 1 = 0$.

f) Find the point of inflexion of curve $x = \log(\dfrac{y}{x})$.

g) If $U_n = \int_0^{\frac{\pi}{2}} \sin^n \theta \, d\theta$ and $n > 1$, then prove that $U_n = \frac{n-1}{n} U_{n-2} + \frac{1}{2}$.

h) Find the length of the arc of the curve
$x = e^{\theta} \sin\theta$
$y = e^{\theta} \cos\theta$
from $\theta = 0$ to $\theta = \frac{\pi}{2}$.

i) Find the area of the segment cut off from the parabola $y^2 = 4x$ by the line $y = x$.

4. Answer any two of the following questions: $2 \times 6 = 12$

a) If $y = \tan^{-1} x$, then prove that
$(1 + x^2)y_{n+1} + 2nxy_n + n(n-1)y_{n-1} = 0$. Hence also find the value of $(y_n)_0$.

b) State and prove Rolle's theorem.

c) State and prove Taylor's with Lagrange's form of remainder.

d) Find the values of a and b in order that $\lim\limits_{x \to 0} \dfrac{x(1 + a\cos x) - b\sin x}{x^3}$ may be equal to 1.

5. Answer any three of the following questions: $6 \times 2 = 12$

a) If $u = \tan^{-1} \dfrac{x^3 + y^3}{x - y}$ prove that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \delta y} + y^2\frac{\partial^2 u}{\partial y^2} = (1 - 4\sin^2 u)\sin 2u\ .$$

b) If $\dfrac{x^2}{a^2 + u} + \dfrac{y^2}{b^2 + u} + \dfrac{z^2}{c^2 + u} = 1$, prove that

$$u_x{}^2 + u_y{}^2 + u_z{}^2 = 2(xu_x + yu_y + zu_z).$$

c) Examine $f(x, y) = x^3 + y^3 - 3axy$ for maximum and minimum value.

d) If ρ_1 and ρ_2 be the radii of curvature at the ends of a focal chord of the parabola $y^2 = 4ax$, then prove that $\rho_1^{\frac{-2}{3}} + \rho_2^{\frac{-2}{3}} = (2a)^{\frac{-2}{3}}$.

e) Find the characteristics of the curve $y(x^2 + 4) = 8$ and then trace it.

6. Answer any two of the following questions: $6 \times 2 = 12$

a) Find the reduction formula for $\int \tan^n x \, dx$ and hence or otherwise find the value of $\int \tan^6 x \, dx$.

b) The circle $x^2 + y^2 = a^2$ revolves round the x-axis, show that the surface area and volume of the whole sphere generated are $4\pi a^2$ and $\frac{4}{3}\pi a^3$ respectively.

c) Change the order of the integration in the double integral $\int_0^{a\cos\alpha} dx \int_{x\tan\alpha}^{\sqrt{a^2-x^2}} f(x,y)\, dx\, dy$.

d) Prove by evaluating the repeated integrals that
$$\int_0^1 dx \int_0^1 \frac{x^2 - y^2}{(x^2 + y^2)^2}\, dy \neq \frac{x^2 - y^2}{(x^2 + y^2)^2}\, dx.$$

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